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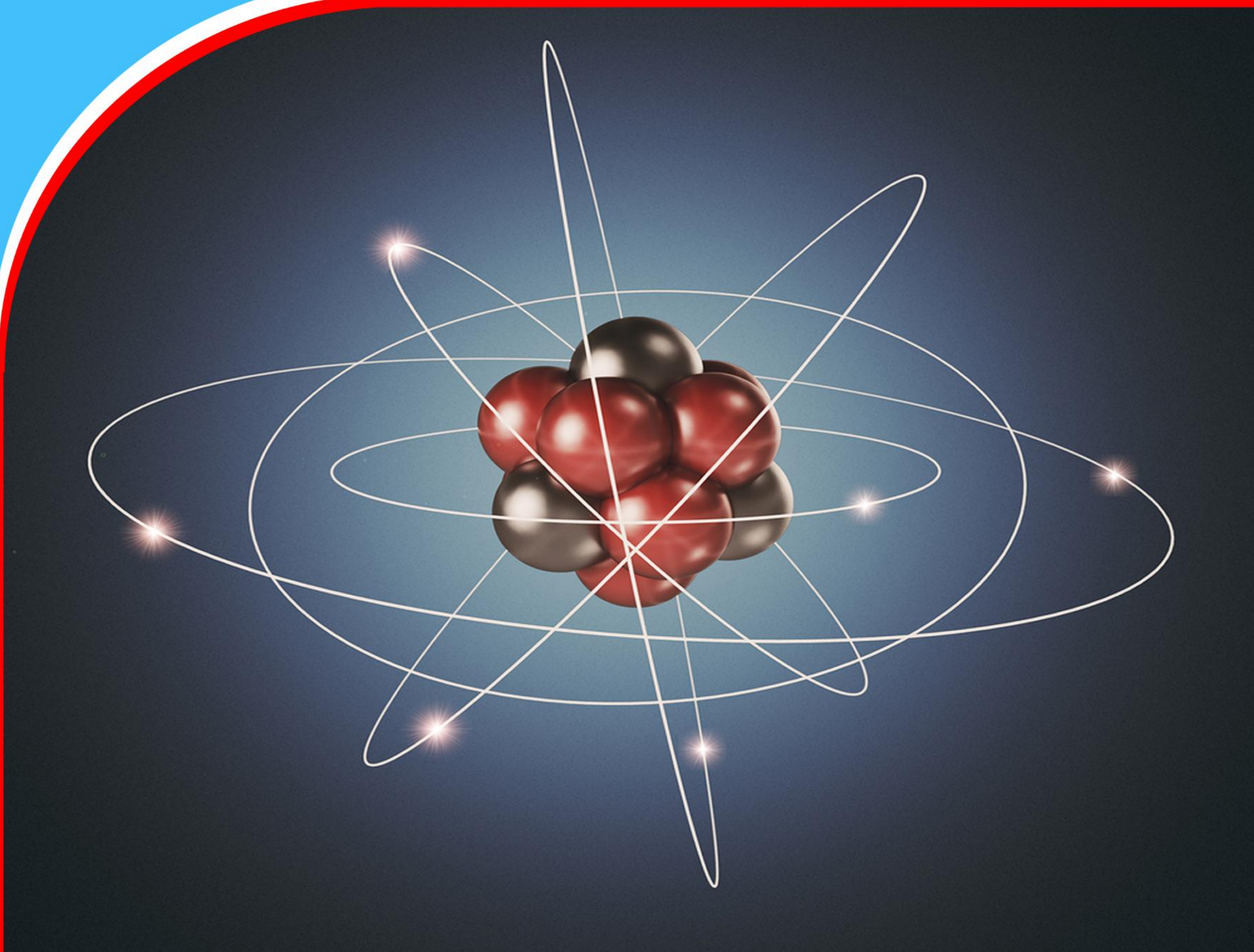

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APPLICATION OF THE DEDUCTIONS FROM NAVIER STOKES EQUATIONS FOR THE DETERMINATION OF FLOW VELOCITY AND THROUGHPUT IN A GAS PIPELINE BY COMPUTATIONAL APPROACH

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Abstract

Theoretical treatment of gas pipeline pressure-flow problem had been presented applying Navier Stokes equation reduced to their appropriate forms by applicable practical conditions. The results obtained from the theoretical analysis tally with the operating conditions of the case study pipelines. The pipelines being Shell Petroleum Development Company (SPDC) and ElfTotal Nigeria Limited. The results obtained by numerical discretization suggested that these pipelines are not optimally operated. Hence, the need to adjust the flow situation to bring pressure and flow throughput to optimal level of performance. Throughput in excess of the operating conditions could be accommodated by these operating pipelines. It is imperative that this could prevent the spread of these vital capital intensive assets. The funds so conserved could be diverted to sourcing for new gas fields to increase the nation's strategic reserves.

Purpose: The purpose of this work is to enable comparative analysis of the results of the deductions from Nervier Stokes equations with that generated by computer simulation of the discrete formulation.

Methodology: Outlining the deductions and developing the discrete formulation. Computer program was developed for the discrete formulation and operational data from operating gas pipelines injected both for the deductions and computational algorithmic coding and the deduced expressions from the Nervier Stokes equations. Results obtained were compared in a bid to address line throughput subject to the operational conditions of the specified gas pipelines in this study.

Findings: The output results of the Nervier Stokes deductions matched closed with operational throughput of the two gas pipelines. The numerical discretization simulation results confirmed that additional throughput far and above 1.8m3/s could still be accommodated by these gas pipelines.

Unique contribution to theory, practice and policy: As earlier predicted, our existing gas pipelines are grossly under-operated. Additional capacity much more than the operational capacity could still be accommodated by these gas pipelines.

Keywords：*Navier Stokes equations; Appropriate forms; Pressure-flow problem; Theoretical treatment; Numerical discretization; Optimal level of performance and Capital intensive assets.*

NOMENCLATURE

ρ--gas density (kg/m^3)

μ—absolute viscosity (Pas)

u—x-component of flow velocity (m/s)

v—r-component of flow velocity (m/s)

i—unit vector in x-direction

j—unit vector in r-direction

k—discrete step in x-direction (m)

h—discrete step in r-direction (m)

M—maximum number of discrete steps in x-direction

N—maximum number of discrete steps in r-direction

$\bar{u}$ – *average flow velocity* (*m* / *s*)

u_{max}—maximum flow velocity (m/s)

Q—flow throughput (m^3/s)

ΔP—line pressure drop (N/m^2)

L—line length (m)

D—nominal pipe diameter (m)

R—pipe radius (m)

r—pipe radial positioning from the center line of the pipe (m)

x—pipe axial positioning along the center line (m)

INTRODUCTION

The ANSI/ASME 31.8 standard code contains empirical set of equations for design, construction and operation of gas pipeline [ANSI/ASME Standard B41.8 (199),APIRPI4C (2001), Shadrack, M. U. &Abam D. P. S. (2013)]. There are other well known gas equations such as compressibility equation, Bernuolii's equation, Euler's equation among others to address gas pipeline pressure-flow-temperature related problems. The deduction from Navierstokes equations are not subject to practical considerations of gas pipeline gas pipeline flow problems but at theoretical parlance. In this regard, it essentially corroborates the findings of practical gas equations be it for optimal or non-optimal approach.

The Navier Stokes deductions for velocity and throughput would be worked upon analytically and numerically using the operating parameters of ofwell known gas companies in Nigeria terrains. It is believed that this would build a network of practical relevance the results of practical flow equations with theoretical models. The work could also be a powerful tool in gas pipeline flow analysis to push a flow to optimal capacity [Abam D. P. S. &Shadrack, M. U. (2013), Adeyanju A. O. &Oyekunle, L. O. (2012].

RELEVANT MODELS

The relevant models as contained in the work titled, "APPLYING DUDUCTIONS FROM NAVIER STOKES EQUATION TO FLOW SITUATIONS IN GAS PIPELINE NETWORK SYSTEM" are as follows :

The expression for pressure gradient and flow velocity in axis-symmetric coordinate system is given as :

$$\rho u\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)=\frac{\partial P}{\partial x} \qquad (1)$$

Integrating equation (1), with respect to r :

$$u=\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2-R^2\right) \qquad (2)$$

The flow throughput is determined by integrating equation (2) with respect to r, thus the expression below is obtained :

$$Q=\int_0^R\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2-R^2\right)2\pi r dr$$

$$=-\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right). \qquad (3)$$

The average flow velocity is expressed as :

$$\bar{u}=\frac{Q}{A}=\frac{R^2}{8\mu}\left(\frac{\Delta P}{L}\right) \qquad (4)$$

To determine the maximum flow velocity and the radial position, $\partial u/\partial r$ is set to zero. Thus,

$$u=\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(r^2-R^2\right). \qquad (5)$$

$$\frac{\partial u}{\partial r}=\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)2r=0$$

$$\therefore \quad r=0$$

$$u_{\max}=\frac{R^2}{4\mu}\left(\frac{\Delta P}{L}\right) \qquad (6)$$

$$u_{\max}=2\bar{u}$$

The expression in equation (5) indicates that the velocity profile is parabolic in nature.

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By numerical discretization approach, equation (1) could be worked upon to obtain expression for the pipeline flow velocity.

$$\rho u \frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial u}{\partial r}\right) = \frac{\partial P}{\partial x} \quad (1)$$

Assuming the flow situation is at steady state, incompressible and fully developed; the continuity equation 1 reduces to the form :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} = 0 \quad (7)$$

Maximum flow velocity occurs at the center line of the pipeline, for this condition to hold, ∂u/∂r=0. This implies that ∂u/∂x=0. Applying these conditions in equation (1) :

$$\mu \frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r}\frac{\partial u}{\partial r} = -\frac{\partial P}{\partial x} \quad (8)$$

Discretizing the expression in equation (8) :

$$\frac{\partial u}{\partial x} = \frac{u_{i,j+1} - u_{i,j}}{\Delta x} + O(\Delta x)$$

$$\frac{\partial u}{\partial x} = \frac{u_{i,j+1} - u_{i,j}}{k} + O(k)$$ forward difference method

$$\frac{\partial u}{\partial r} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta r} + O(\Delta r)$$

$$\frac{\partial u}{\partial r} = \frac{u_{i+1,j} - u_{i-1,j}}{2h} + O(h)$$ central difference method

$$\frac{\partial^2 u}{\partial r^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta r^2} + O(\Delta r^2)$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + O(h^2)$$

Where, $\Delta x = k$ *and* $\Delta r = h$.

Substituting for the various terms in equation (8) :

$$\mu \frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r}\frac{\partial u}{\partial r} = \frac{\partial P}{\partial x} \qquad (8)$$

$$\mu\left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2}\right) + \frac{\mu}{r_i}\left(\frac{u_{i,j+1} - u_{i,j}}{h}\right) = \left(\frac{\Delta P}{L}\right)$$

$$\mu\left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h^2}\right) + \frac{\mu}{ih}\left(\frac{u_{i,j+1} - u_{i,j}}{h}\right) = \left(\frac{\Delta P}{L}\right)$$

$$Att \;\; pipe \;\; center \;\; line, \; r = 0, u = u_{\max} \;\; and \;\; \frac{\partial u}{\partial r} = 0.$$

$$\therefore \;\; \frac{\partial u}{\partial r} = \frac{u_{i,j+1} - u_{i,i}}{h} = \frac{u_{i,i} - u_{i,j-1}}{h} = 0$$

$$u_{i,j+1} = u_{i,i} \;\;\; and \;\;\; u_{i,i} = u_{i,j-1}$$

$$\frac{\partial P}{\partial x} = -\frac{\Delta P}{L}$$

$$u_{i,j+1} = \left[\left(\frac{1}{\mu}\right)\left(\frac{\Delta P}{L}\right) + \left(\frac{1}{jh^2} + \frac{1}{h^2}\right)u_{i,i}\right] / \left(\frac{1}{jh^2} + \frac{1}{h^2}\right) \qquad (9)$$

$$i = M,, M - 1,, M - 2.......1$$

$$j = 1,2,3..........................N$$

$$u(i, N) = u(1, j) = 0 \qquad boundary \;\;\; conditions$$

Determination of Flow velocity Analytically :

(i) Shell Petroleum Development Company Gas Pipeline

The flow throughput and velocity are determined as follows applying the operating data of the pipeline :

Upstream pressure, P_1=81bar

Downstream pressure, P_2=63bar

Line length, L=116km(116000m)

Pipe nominal diameter, D=36”(0.9144m)

Operational Throughput=1.8m^3/s

Gas viscosity, μ=0.156Pas

Gas density, ρ=80kg/m^3

Applying equation (3) to determine the line throughput

$$Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$
$$= \frac{\pi R^4}{8\mu}\left(\frac{\Delta P}{L}\right)$$
$$= \frac{\pi D^4}{128\mu}\left(\frac{\Delta P}{L}\right) \quad (3)$$
$$= \frac{\pi (0.9144)^4}{128 \times 0.156}\left(\frac{81-63}{116000}\right) \times 10\text{^}5 = 2.04 m^3/s$$

The average gas flow velocity is obtained from equation 4.

$$\bar{u} = \frac{Q}{A} = \frac{R^2}{8\mu}\left(\frac{\Delta P}{L}\right) \quad (4)$$
$$= \frac{(0.9144/2)^2}{8 \times 0.156}\left(\frac{81-63}{11600}\right) = 2.6 m/s$$

(ii)ElfTotal Nigeria Limited Gas Pipeline

Upstream pressure, P_1=84.1bar

Downstream pressure, P_2=63bar

Line length, L=134km(134000m)

Pipe nominal diameter, D=36"(0.9144m)

Operational Throughput=1.8m^3/s

Gas viscosity, µ=0.148Pas

Gas density, ρ=73.24kg/m^3

Applying equation (3)to determine the line throughput

$$Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial P}{\partial x}\right)$$
$$= \frac{\pi R^4}{8\mu}\left(\frac{\Delta P}{L}\right)$$
$$= \frac{\pi D^4}{128\mu}\left(\frac{\Delta P}{L}\right) \quad (3)$$
$$= \frac{\pi(0.9144)^4}{128\times 0.148}\left(\frac{84.1-63}{134000}\right)\times 10\text{^}5 = 1.82 m^3/s$$

The average gas flow velocity is obtained from equation (4).

$$\bar{u} = \frac{Q}{A} = \frac{R^2}{8\mu}\left(\frac{\Delta P}{L}\right) \quad (4)$$
$$= \frac{(0.9144/2)^2}{8\times 0.148}\left(\frac{84.1-63}{134000}\right) = 2.78 m/s$$

(ii) Determination of Flow velocity By Numerical Discretization Method :

The programming algorithmic coding goes thus:

```
% Computer Simulation of Discretize Navier Stokes models
% I--subscript for radial steps in r-direction
% J--subscript for radial steps in x-direction
%  INITIALIZATION
%  Upsream pressure, P1(N/m2)
%  P1=81*10^5;
   P1=84.1*10^5;
%  Dostream pressure, P2(N/m2)
  P2=63*10^5;
%  Overall line pressure drop, Dp(N/m2)
  DP=P1-P2;
%  Line length,  L(m)
%  L=116000;
   L=134000;
% Nominal pipe diameter, D (m)
```

```
  D=0.9144;
% Gas throughput, Q (m3/s)
  Q=1.8;
% pipe crossectional area, A (m2)
  A=(pi*D^2)/4;
% Gas flow velocity, V (m/s)
  V=Q/A;
% Gas desityGD(kg/m3)
  DG=73.24;
%  Absolute gas viscosity,  GV(Pas);
% Shell Petroleum Development Company Absulte Gas Viscosity, GV
(Pas)
%  GV=0.156;
% ElfTotal Nigeria Limited Absulte Gas Viscosity, GV (Pas)
   GV=0.148;
%  M--Maximum iteration steps in r-direction
   M=4;
%  N--Maximum iteration steps in x-direction
   N=4;
%  h--Discrete interval in r-direction
    k=0.2286;
%  k--Discrete interval in r-direction
    h=0.2286;
for I=1:1:4
for J=4:-1:1
U(I,N)=0;
U(1,J)=V;
      U(I,J+1)=((((DP/L)*(1/GV))+(U(I,J)*(h^(-2)+(J^(-1)*h^(-
2))))))/(h^(-2)+(J^(-1)*h^(-2))));
disp('I               J')
fprintf('%12.4f\n',I,J,U(I,J+1))
end
end
```

Using the Shell Petroleum Development Company Nig. Ltd operational data flow velocity within the limit of 2.599 to 10.8165m/s was confirmed. The operational data for ElfTotal Nig. Ltd gas pipeline produced flow velocity in the range of 2.78 to 10.8165m/s.

ANALYSIS OF RESULTS

Though ElfTotal and Shell operational throughput was 1.8m^3/s, but within a reasonable limit there is a measure of closeness between Navier Stokes prediction through analytical approach and the operational throughput of the case study pipelines. This was subject to the fact that Shell computational throughput is 2.04m^3/s and that of ElfTotal is 1.82m^3/s. Numerical computational approach predicted flow velocity within the limit of 2.78 to 9.963 m/s for Shell operational data and 2.78 to 10.2344 m/s for ElfTotal. The results are indications that these operating pipelines are under-operated. Hence, the need to operate at optimal level of performance.

RECOMMENDATION

The findings in this work confirmed that operating gas pipelines in Nigeria terrains are under-operated. Future work should be geared towards developing empirical models for gas pipeline design and installation concepts while addressing optimality in performance as the potent criteria.

CONCLUSION

Navier Stokes equations had been reduced to the appropriate forms to handle gas pipeline pressure-flow problems by computational approach. It was discovered that the prediction of theoretical flow equations agreed with the present operating conditions of Nigeria operating pipelines. The numerical computations suggest even the need to push the performance scenario to optimality.

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