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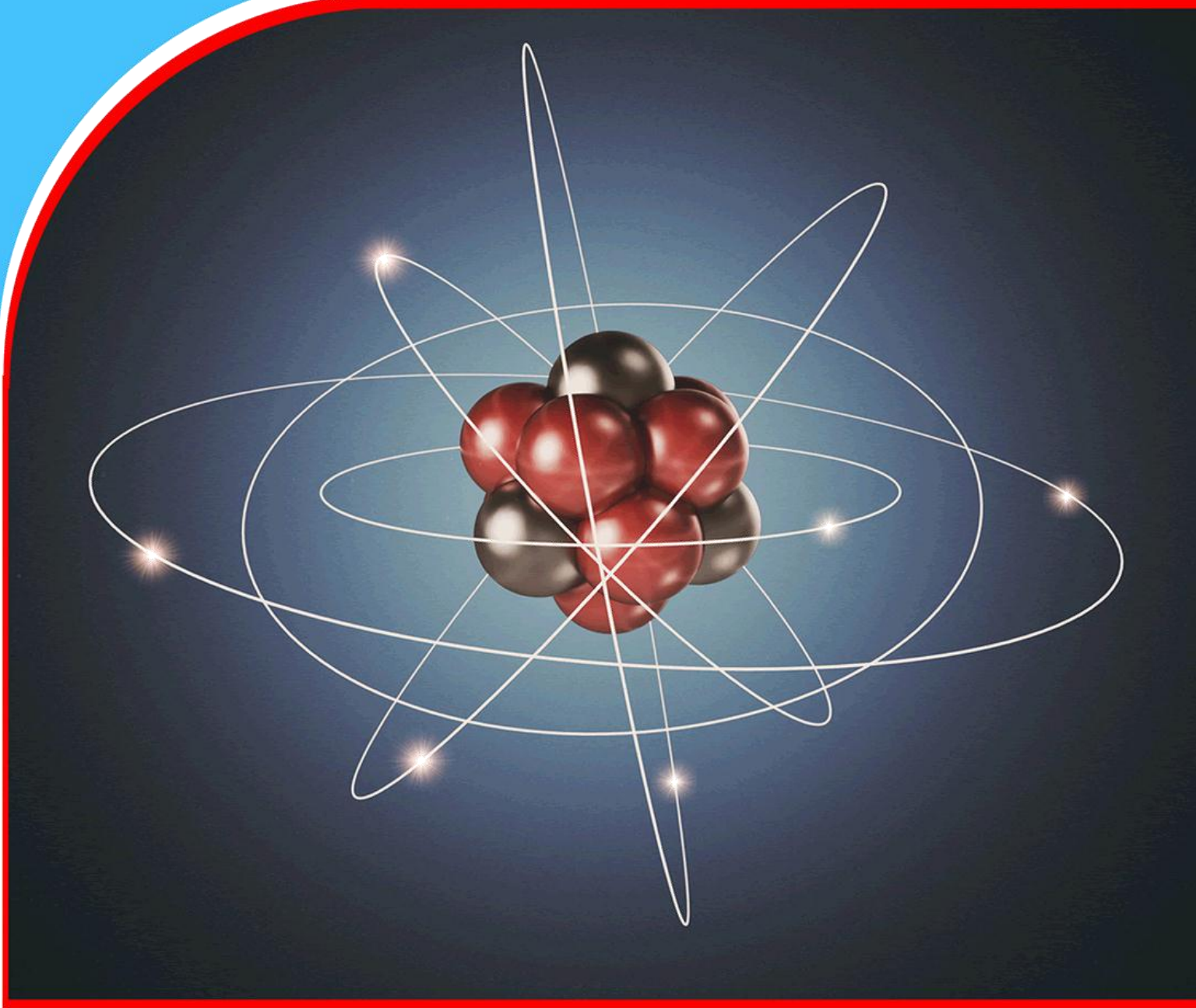

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Abstract

The structure of some even-even Cd isotopes was studied using the interacting boson model-2 (IBM-2). For some selected transitions, the theoretical calculation of energy levels was performed; the transition probabilities and multipole mixing ratios were obtained, and compared with experimental results. The set of model parameters used in this study indicates that most of the isotopes of Cadmium are vibrational. As a function of the neutron number across the transitional region, notably χ, κ the behaviors of the set of parameters in the Hamiltonian were studied, which clearly indicated the change of shape of these isotopes from vibrational to almost rotational.

Keywords: *Interacting Boson Model, Energy Levels, B(E2), (E2/M1)*

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Introduction

In quadrupole correlations in nuclei, the neutron-proton interaction is considered to play a dominant role. As a consequence, the excitation energies in nuclei near a closed shell of collective quadrupole excitations are strongly dependent on the number of nucleons outside the closed shell. The isotopes $^{110\text{-}116}$Cd (Z=48), with the number of neutrons varying from 62 to 68, are known to display $N_\pi = 1$ and N_ν varying from 6 to 9. The transitional region that happened at the bottom limit of the deformed nuclei set is where the Cd isotopes lie.

To describe the collective properties of many medium nuclei, the interacting boson approximation is very efficient. Arima and Iachello present the interacting boson model (IBM) [1-4]. To describe the collective properties of many medium nuclei, the interacting boson approximation has been very efficient. According to Arima and Iachello [1-4] and Casten [5] interacting boson model (IBM) has been generally accepted as a tractable theoretical framework for correlating, explaining and predicting the collective low-energy properties of complex nuclei. The low-energy states of even -even nuclei in terms of interactions between *s* ($L = 0$) and *d* ($L = 2$) bosons are described in this model. By using somewhat powerful and efficient group theory methods, the corresponding Hamiltonian is diagonalized in this boson space.

The Interacting Boson Model-2 (IBM-2)

For a given nucleus, the boson numbers N_ν and N_π are found by counting neutrons and protons from the nearest closed shells. The vector space of IBM-2 is then just the product of all possible states $(s, d)^{N\nu}$ with those of $(s, d)^{N\pi}$, where in each factor the set of states is the same as in IBM-1 [6]. In this analysis used the following Hamiltonian [7]. The boson numbers N_ν and N_π are identified for a given nucleus by counting neutrons and protons from the nearest closed shells. IBM-2 vector space is then merely the product of all possible states (s_π, d_π) with those of (s_ν, d_ν), where the set of states is the same in each factor as in IBM-1[6]. We used the following Hamiltonian [7] in this analysis:

$$H = \varepsilon(\tilde{n}_{d_\nu} + \tilde{n}_{d_\pi}) + \kappa.Q_\nu.Q_\pi + \tilde{\kappa}(Q_\nu.Q_\nu + Q_\pi.Q_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad \ldots\ldots(1)$$

Where ε is the d-boson energy, κ is the strength of the quadrupole interaction between neutron and proton bosons. In the IBM-2 model, the quadrupole moment operator is given by [8]:

$$Q_\rho = (s_\rho^+ \tilde{d}_\rho + d_\rho^+ s_\rho)^{(2)} + \chi_\rho (d_\rho^+ d_\rho)^{(2)} \quad \ldots\ldots(2)$$

Where $\rho = \nu, \pi$, χ_ρ is the quadrupole deformation parameter for neutrons ($\rho = \nu$) and protons ($\rho = \pi$). The last term $M_{\nu\pi}$ is the Majorana interaction, which has the form:

$$M_{\nu\pi} = \tfrac{1}{2}\xi_2 (s_\nu^+ d_\pi^+ - d_\nu^+ s_\pi^+)^{(2)}.(\tilde{s}_\nu \tilde{d}_\pi - \tilde{d}_\nu \tilde{s}_\pi)^{(2)} - \sum_{k=1,3} \xi_k (d_\nu^+ . d_\pi^+)^{(k)}.(\tilde{d}_\nu . \tilde{d}_\pi)^{(2)} \quad \ldots\ldots(3)$$

The term $\tilde{\kappa}(Q_\nu.Q_\nu + Q_\pi.Q_\pi)$ is a quadrupole interaction among similar bosons. This part of the interaction introduces a triaxial component into the IBM-2 Hamiltonian when χ_ν and χ_π have opposite signs. This is the main deference between this Hamiltonian and the usual IBM-2 Hamiltonian.

$$H = \varepsilon(\tilde{n}_{d_\nu} + \tilde{n}_{d_\pi}) + \kappa.Q_\nu.Q_\pi + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad \ldots\ldots(4)$$

Where the terms $V_{\nu\nu}$ and $V_{\pi\pi}$ are the neutron - neutron and proton - proton d-boson interactions only

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Electromagnetic Transitions and Quadrupole Moments

The general one-body E2 transition operator in the IBM-2 is:

$$T(E2) = e_\nu.Q_\nu + e_\pi.Q_\pi \quad \ldots\ldots\ldots\

The Energy Levels

In general, the energy levels spacing behavior is viewed as undergoing a transition from pure vibrational SU (5) limit to nearly rotational SU (3) limit. The B(E2) values for a few transitions in the Cd isotopes are calculated. The effective charge was calculated using equation (6). The calculated energy levels compared with experimental results [11-14] are given in table 2.

Table 2: The IBM-2 calculation and experimental energy levels of ^{110,112,114,116}Cd.

^{110}Cd			^{112}Cd			^{114}Cd			^{116}Cd		
J^π	Exp. (MeV)	IBM-2 (MeV)	J^π	Exp. (MeV)	IBM-2 (MeV)	J^π	Exp. (MeV)	IBM-2 (MeV)	J^π	Exp (MeV)	IBM-2 (MeV)
0_1^+	0.000	0.000	0_1^+	0.000	0.000	0_1^+	0.000	0.000	0_1^+	0.0000	0.000
2_1^+	0.658	0.801	2_1^+	0.618	0.743	2_1^+	0.588	0.711	2_1^+	0.658	0.801
0_2^+	1.340	1.330	0_2^+	1.224	1.221	0_2^+	1.134	0.961	0_2^+	1.283	0.832
4_1^+	1.542	1.541	4_1^+	1.415	1.435	2_2^+	1.210	1.227	4_1^+	1.542	1.541
2_2^+	1.783	1.641	2_2^+	1.312	1.515	4_1^+	1.284	1.284	2_2^+	1.783	1.641
6_1^+	2.479	2.260	2_2^+	1.469	1.776	0_3^+	1.305	1.381	2_3^+	2.479	2.260
4_2^+	2.250	2.418	6_1^+	2.167	2.172	3_1^+	1.864	1.713	4_2^+	2.250	2.418
3_1^+	2.162	2.475	0_3^+	1.433	2.162	2_3^+	1.364	1.742	3_1^+	2.162	2.475
3_2^+	2.433	2.491	4_2^+	2.081	2.252	4_2^+	1.732	1.745	4_3^+	2.433	2.491
4_3^+	2.561	2.508	3_1^+	2.637	2.298	2_4^+	1.841	1.824	2_4^+	2.561	2.508
5_1^+	2.926	3.055	4_3^+	2.457	2.376	6_1^+	2.400	1.835	2_5^+	2.926	*3.055*
6_2^+	2.876	3.132	8_1^+	2.880	2.819	2_5^+	2.048	2.100			
6_3^+	3.122	3.175				4_3^+	2.391	2.158			
						3_2^+	2.205	2.214			

The low lying energy levels calculated by the IBM-2 compared with the experimental data [11-14] are given in figure 1.

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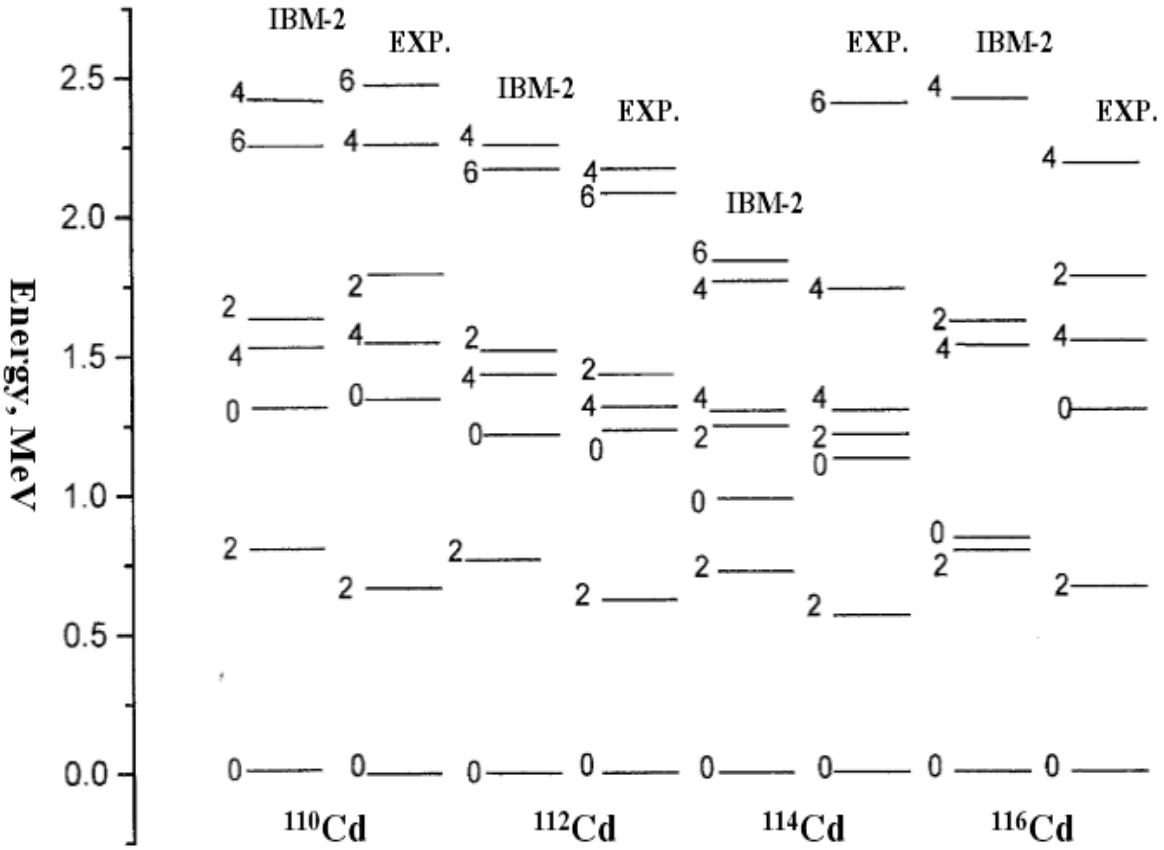

Figure 1: Comparison between IBM-2 results and experimental values for Cd isotopes

Table 3 shows the results of these calculations and a comparison with the experimental data. There is no real deviation between theory, experimental [15] and previous work [16].

Table 3: The comparison between IBM-2 and experimental electric transition probabilities for ^{110,112,114,116}Cd isotopes.

Isotopes	$E_\gamma(MeV)$	Transition	IBM-2	Exp
^{110}Cd	0.6578	$2_1^+ \rightarrow 0_1^+$	0.1031	0.0934±0.0037
	1.1252	$2_2^+ \rightarrow 2_1^+$	0.2676	0.101±0.029
	0.8842	$4_1^+ \rightarrow 2_1^+$	0.1486	0.143605±0.02340
^{112}Cd	0.6176	$2_1^+ \rightarrow 0_1^+$	0.1048	0.0934±0.0037
	0.6944	$2_2^+ \rightarrow 2_1^+$	0.3258	0.1048±0.002793
	1.4634	$4_1^+ \rightarrow 2_1^+$	0.1810	0.19904±0.023044
^{114}Cd	0.5585	$2_1^+ \rightarrow 0_1^+$	0.1241	0.1152±0.0046
	0.6512	$2_2^+ \rightarrow 2_1^+$	0.3272	0.0812±0.0227
	0.7258	$4_1^+ \rightarrow 2_1^+$	0.1830	0.1106±0.046

^{116}Cd	0.5840	$2_1^+ \rightarrow 0_1^+$	0.1151	0.1162±0.0046
	0.6310	$2_2^+ \rightarrow 2_1^+$	0.3260	0.073±0.02
	0.1000	$4_1^+ \rightarrow 2_1^+$	0.18406987	0.1968±0.04956

The Multipole Mixing Ratio δ(E2/M1)

The calculated multipole mixing ratios values are given in table 4 together with the experimental data. In general, there is a clear difference between calculation and experiment in sign and magnitude. The change in sign is sometimes observed in M1 and takes place when the E2 matrix element is small.

Table 4: The E2/M1 mixing ratios for ^{110,112,114,116}Cd isotope

Isotope	Transition Energy E_i (keV)	Spin Parity $I_i \longrightarrow I_f$	IBM-2 δ(E2/M1)	Experimental δ(E2/M1)
^{110}Cd	1542	$2_2^+ \rightarrow 2_1^+$	–6.023	$-(1.5^{+0.9}_{-0.4})$
^{112}Cd	1312	$2_2^+ \rightarrow 2_1^+$	–1.402	$-(3.5^{+0.9}_{-0.8})$
^{114}Cd	1210	$2_2^+ \rightarrow 2_1^+$	+0.815	$-(1.5^{+0.9}_{-0.5})$
^{116}Cd	1213	$2_2^+ \rightarrow 2_1^+$	–2.600	$-(1.5^{+0.9}_{-0.5})$

Conclusion

The researchers presented the results of a systematic study of four even-even Cadmium isotopes in the framework of the IBM-2. These nuclei are viewed as undergoing a transition from SU(5) limit to nearlySU(3) limit.

The results obtained are:

(1) The low lying energy levels for ^{110,112}Cd behave like vibrational limit (equispace); while in ^{114,116}Cd behave like rotational limit.

(2) The behavior of the parameters indicates that the nuclei shapes change as function of neutron number.

(3) The calculated B(E2) values show no essential contradiction to the experimental data

(4) The calculation finds large δ-mixing ratio for ^{114}Cd as in the experimental results

(5) The parameters which have a great effect the most appear are $\varepsilon d, k,$ and X_ν.

References

[1] F. Iachello and A. Arima, *The Interacting Boson Model*, (Camb. Univ. Press, (1987).

[2] A. Arima and F. Iachello, Ann. *Phys. NY*, 99 (1976) 253.

[3] A. Arima and F. Iachello, Ann. *Phys. NY*, 111 (1978) 201.

[4] A. Arima and F. Iachello, Ann. *Phys. NY*, 123 (1979) 468.

[5] R. F. Casten, *Nucl. Phys.*, A 347 (1980) 173.

[6] J. P. Elliott, C. M. Snowden, *Rep. Prog. in Phys*, 48 (1985) 171.

[7] T. Tagziria, M. Elahrash, W. D. Hamilton, M. Finger, J. John, and V. N. Pavlov, *J. Phys. G: Nucl. Part. Phys.*, 16 (1990) 1323.

[8] S. A. Berendakov, L. I. Gover and A. M. Demidov, *Physics of Atomic Nuclei,* 61 (1998) 1437.

[9] Y. X. Liu, G. L. Long and H. Z. Sun, *J. Phys. G: Nucl. Part. Phys.*, 17 (1991) 877.

[10] T. Otsuka and N. Yoshida, *the IBA-2 computer program NPBOS*, University of Tokyo (1985).

[11] A. M. Demidov, S. Y. Araddad, S. M. Sergiwa, G. A. M. Rateb, V. A. Kurkin, S. M. Zleetni, A. M. Shermit, *Izv. Akad. Nauk USSR (Ser. Fiz.)* 56 (1992) 1.

[12] S. Y. Araddad, A. M. Demidov, M. M. Dufani, V. A. Kurkin, G. A. M. Rateb, S. M. Zleetn, I. V. Mikhailov I. V., *the Topics in Atomic Science and Technology, Series Nuclear Constants No. 1,* Moscow, (1991) 47.

[13] S. Y. Araddad, A. M. Demidov, V. A. Kurkin, G. A. M. Rateb, S. M. Zleetni, O. K. Zhuravlev, *Yadernaya Fisika*, 46 (1987) 40.

[14] M. S. Elahrash, S. Y. Araddad, A. M. Demidov, S. M. Sergiwa, M. M. Dufani, V. A. Kurkin, G. A. M. Rateb, S. M. Zleetni, A. M. Shermit, *Proceedings of the First Arab Conference on the Peaceful Use of Atomic Energy, Tripoli -Libya,* (1992).

[15] W. T. Milner, F. K. McGowan, P. Stelson, R. L. Robinson and R. O. Sayer, *Nucl.Phys.* A129 (1969) 687.

[16] V. P. Varsheny, K. K. Gupta, A. K. Chanbey and D. K. Gupta, J.*Phys.G:Nuc.Phys.*, 8 (1982) 727.