CH 1 5

Discounting and Accumulating

\[ \delta(t) = \begin{cases} 
\delta_1(t) & 0 < t \leq t_1 \\
\delta_2(t) & t_1 < t \leq t_2 \\
\delta_3(t) & t > t_2 
\end{cases} \]

Accumulated value at time \( t \) of a pmf of 1 at time 0 is
MODERNIZING PENSION FUND CONTRIBUTION

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Abstract

Purpose: The purpose of this research is to develop a pension model which uses mobile phone technology to facilitate contribution towards the user’s pension scheme.

Methodology: The entry age pension funding method, and annuity present value & accumulations methods are used to come up with the contribution rate and the actuarial liability for the model.

Findings: The model developed is independent of salary or any other income and only takes into consideration those legally registered to use mobile phone. Being a private pension plan, and the fact that it involves the member directly via and individual mobile phone subscription, wider coverage is anticipated.

Unique contribution to theory, practice and policy: Contribution towards pension scheme by use of mobile phone technology in which contribution (which is fixed and independent of salary) is achieved by how frequently, the credit is loaded and used by the subscriber.

Keywords: Pension plans, mobile phone technology, entry age, annuities.
1.0 Introduction

Saving for future use is one of the best practices which is encouraged by any other caring government. There are several ways of saving (in Kenya) which include the following; mandatory savings imposed by governments (e.g. National Social Security Fund-NSSF, voluntary savings (still with NSSF, and Saving for oneself in a financial institution), taking insurance Policies and many more.

Mandatory Savings by the government (NSSF) has the majority of contributors going by the fact that it is mandatory to both government and private employees. NSSF is contributed by both employee and the employer, with the minimum contribution being 200 Kenyan Shillings. On the other hand, voluntary savings is strongly encouraged by the government through the NSSF. It came into existence due to the fact that not everyone is employed and an individual may decide to contribute for future use from his source of income, be it informal employment and/or from a business. Voluntary savings also include saving for one-self in a financial institution. One can also save from business, part of salary or employment proceeds by a financial institution for example banks, insurance companies, SACCOs (equities)

In this paper, a flexible and universal ways of saving is suggested. This is called digital savings or more formally Save As You Use (SAYU). With the widespread and universal use of technology in the day to day activities, saving using such technologies is key in the present world driven by technology. Mobile phones have become an integral part of our daily activities right from communication, financial transactions, education, and many more. Nevertheless, we have always been curious when it comes to the future benefits which the user gains as a result of using such technological device. One such benefits have been observed in Tanzania whereby traditional women groups have been replaced with a digital savings technique in which numerous benefits have been achieved as indicated in [9]. There has been increased security of funds and trust in handling such finances. Acquiring loans, savings, and other transactions can easily be done by the said technology. Thus it can be concluded as indicated in [8] that mobile phones have contributed immensely in the economic development not only in Africa but also globally.

Mobile transactions come with cost from end-to-end users and we suggest that some small percentage of this cost also be transformed into saving and hence the term digital saving. During the period of Corona Virus (Covid-19) an extra percentage of credit (airtime) was issued by the three telecommunication giants in Kenya (Telekom, Safaricom, and Airtel) upon purchase. This tax relief was part of the efforts to caution the public from the effects of the pandemic. A closely related approach is suggested in the present research to help individuals save some money as a result of using mobile phones in communications and any other financial transaction [5], [6], and [7].

This research has been driven by the fact that most researchers give alot of attention to the development of statutory pensions as compared to private pension plans as indicated in [1]. Metzger (2007) in [3] observed that there is a greater possibility for additional savings by private pensions for retirement with increased rate as a supplement to the statutory pensions scheme in Germany.
1.1 Statement of the Problem

Contribution towards pension has been a very big nightmare to most of the citizens particularly in less developed countries for example Kenya. This has been attributed to the levels of poverty that is ravaging such nations, a situation which is directly connected to unemployment and lack of proper income. This has made the contribution towards pensions or simply general savings to be left particularly to those who have formal employment or those who do some businesses like Small and Medium size entrepreneurs. It can therefore be observed that majority of the citizens have no access to such contribution mechanisms. However, a greater fraction of the general population has access to mobile phone, a scenario which has been brought about by the advancement in mobile technology and its associated products like Mpesa, which has facilitated the transfer of funds from one user to another globally. Our research leverages on the widespread use of mobile technology to aid in the contribution towards pension or savings by the citizens.

1.2 Objective of the Study

The main objective of this research is to develop a mathematical model which is used to help users to contribute towards pension or savings for future use by help of mobile technology, precisely how frequently does a subscriber uses (loads and consumes airtime) his/ her mobile phone?

2.0 THE MODEL

The main pension funding methods are classified broadly into two major methods which include prospective and accrued benefit funding. The former and the latter methods are each further classified into attained age (AA) & entry age (EA), and projected unit & current unit methods respectively as indicated in [4] and [10]. In this paper, the Entry Age method is used to come up with the present model, due to the fact that it targets a stable contribution rate resonates with the current model. It is also important to mention that for a user to start saving, one needs to have obtained a national identity card to facilitate the process. For example, the standard contribution rate for Entry Age method (EASCR) for a benefit scheme that provides a pension at retirement based on the final earnings and uniform accrual is given by,

\[ EASCR = \frac{(R-E) \times S}{A} \times (1+i)^{R-E} \times a_R^n \times \frac{\left(1+\frac{e}{1+i}\right)}{a_R^{R-E}} \]  

(1)

Where

- R is the assumed retirement age
- E is the assumed entry age
- S is the salary (or earnings) at the date of valuation
- A is the rate of pension accrual
- e is assumed annual earnings (or salary) growth
- i is the discount rate
- \( a_R^n \) is the value of an annuity payable from age R, based on the assumed proportion of male and females in the scheme (allowing for any contingent spouse’s pension and pension increases)
• $a_{R=E|}$ is an annuity to determine the present value of earnings over all service for a new entrant.

In any pension funding methods, the standard contribution rate is normally given by the present value (PV) of the benefits accruing over future membership divided by the PV of future earnings. In this paper, the main focus is to determine the standard contribution rate by use of mobile technology as opposed to salary earnings. Therefore, to come up with the present model, we need to define how an individual contributes to the pension fund for his/her own future benefits.

The EAAL for an individual member can be expressed as

$$\frac{(P+F)}{A} \times S \times \left(\frac{1+e^\delta}{1+i}\right)^F \times a' - SCR \times S \times a_{E|}$$ \hspace{1cm} (2)

Where

• $P$ is the past service at date of valuation
• $F = (R - x)$ is the future service

2.1 Estimating Monthly Credit Consumption (CCM)

The total monthly credit consumption (CCM) in the present model is synonymous to contributions which are independent of salary, are fixed to a monthly sum of Ksh. CCM, payable continuously as indicated in [4]. Therefore, its definition must be explicit to enhance achievement of the main objective. Every individual loads and consumes his/her own credit differently, however, the formula is invariant. We assume for simplicity of the model that an individual contributes a daily constant amount of $C$ towards the scheme. This results to sum of series of regular daily payments to obtain the CCM.

By definition the value of an accumulation of series of regular payments is given by

$$s^{\delta}_{\bar{t}|} = \int_0^t e^{\delta s} ds = \begin{cases} \frac{t}{e^{\delta t-1}} & \text{if } \delta > 0 \\ t & \text{if } \delta = 0 \end{cases}$$ \hspace{1cm} (3)

On discounting Equation (3), we obtain

$$e^{-\delta t} \times s^{\delta}_{\bar{t}|} = a^{\delta}_{\bar{t}|} = \begin{cases} t & \text{if } \delta = 0 \\ \frac{1 - e^{-\delta t}}{\delta} & \text{if } \delta > 0 \end{cases}$$ \hspace{1cm} (4)

Thus from equation (3), the Monthly Credit Consumption is given by

$$CCM = C \times s^{\delta}_{30|}$$ \hspace{1cm} (5)

Finally, the PV of Monthly Credit consumption is given by

$$PV(CCM) = \left(C \times s^{\delta}_{30|}\right) \times e^{-\delta(30)} = CCM \times e^{-\delta(30)} \hspace{1cm} \delta > 0$$ \hspace{1cm} (6)

Where $\delta$ is any appropriately chosen daily force of interest.
Consequently, a constant CCY is introduced, which denotes the yearly Credit Consumption by an individual subscriber. It is important to note that the standard contribution rate takes care of future benefits that accrue over the stipulated period.

### 2.2 The Mean PV of Future Contribution

It is assumed that contributions are fixed at an annual sum of CCY payable continuously. In this paper, it is also assumed for simplicity that at age $y$, the subscriber would be able to earn a benefit.

The mean present value of future contributions for the present model is given by

$$\int_0^{y-x} V^t \frac{l_{x+t}}{l_x} dt \approx CCY \sum_{t=0}^{y-(1+x)} V^{t+\frac{1}{2}} \frac{l_{x+t+\frac{1}{2}}}{l_x}$$

In terms of Commutation Functions,

$$CCY \sum_{t=0}^{y-(1+x)} D_{x+t} \approx CCY \sum_{t=0}^{y-(1+x)} \frac{D_{x+t}}{D_x} = CCY \left( \frac{\bar{N}_x}{D_x} \right)$$

Since,

$$\sum_{t=0}^{y-(1+x)} D_{x+t} = \bar{N}_x$$

### 2.3 The Standard Contribution Rate of the Present Model

The standard contribution rate (SCR) is normally given by the PV of the benefits accruing over future membership divided by PV of future earnings. Therefore, the standard contribution rate for an individual aged $x$ using mobile technology as opposed to salary earnings is given by

$$SCR = \frac{CCY \left( \frac{\bar{N}_x}{D_x} \right)}{CCY \times (y-x)} = \frac{\left( \frac{\bar{N}_x}{D_x} \right)}{(y-x)}$$

where $a_{y-x}$ is the present value of a life annuity.

### 2.4 The Actuarial Liability of the Present Model

The Actuarial Liability (AL) of the scheme is the difference between the PV of total benefits, based on the projected final earnings for active subscribers and the SCR multiplied by the PV of the total projected credit consumption for all active subscribers throughout the expected future subscriptions [4] and [10].

Algebraically,

$$AL = \left( CCY \times \left( \frac{\bar{N}_x}{D_x} \right) \times a_{y-x} \right) - \left( SCR \times CCY \times a_{y-x} \right)$$

where,

- CCY is the yearly credit consumption;
- $(y-x)$, is the total active subscription period;
SCR is the standard contribution rate obtained in Equation (7).

3.0 Summary, Conclusions, and Recommendation

3.1 Summary of Findings and Conclusion

Benefits of the suggested model include the following; nearly all the legal subscribers to any of the three telecommunication companies will be able to save and consequently this will provide a wide coverage hence more citizens will be able to invest in their future lives, and secondly those who have no any other savings strategies will have a platform, the daily percentage saved will be a small fraction which will never be a burden to the subscriber, and finally, the percentage of the country’s saving will certainly rise. This is also observed in [2] that “participants in private pension plans are greater savers with respect to both past savings and expected future savings than non-participants”.

3.2 Recommendation

The model assumes that the amount contributed by each subscriber is constant yet it varies per individual subscriber, and of course the daily subscriptions also varies greatly. Therefore, the model is open to improvement to take into consideration the variations across the subscribers. The model should also take into consideration the effects of investments.

References

[9] Digital Savings Group pilot report April 2018