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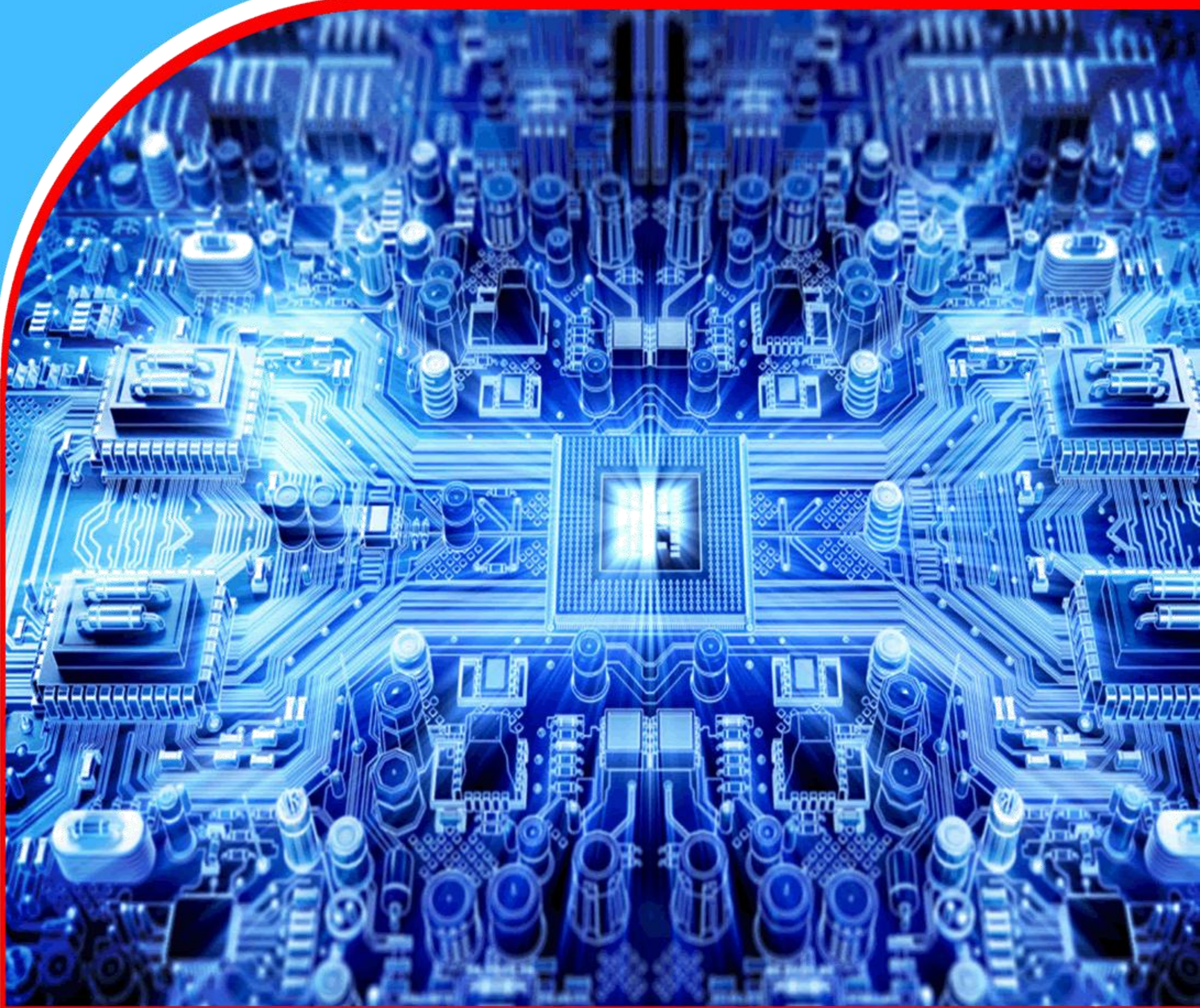

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Comparison Between Scheffe's Second Degree (5,2) And Third Degree (5,3) Polynomial Models In The Optimization Of Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC)

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ABSTRACT

Purpose: This research work aimed at formulating an optimization model based on Scheffe's Third Degree Polynomial (5,3) that can be used to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC), which is then compared to Scheffe's Second Degree Polynomial (5,2) formulation developed by Nwachukwu and others (2017) .

Methodology: Using Scheffe's Simplex method, the compressive strength of GFRC was determined for different ratios. Control experiments were also carried out and the compressive strength determined. After the tests have been conducted, the adequacy of the model was tested using fisher's f-test and the result of the test shows a good correlation between the model and control results.

Findings: Optimum compressive strength for the Scheffe's (5,3) model was obtained as 21.82 N/mm^2. This is slightly higher than the optimum compressive strength for Scheffe's (5,2) model which was obtained as 20.71 N/mm^2 by Nwachukwu and others (2017). Since structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa (or 20 to35 N/mm^2), it then means that optimized GFRC based on both Scheffe's models can produce the required compressive strength needed in major construction projects such as bridges and light-weight structures.

Recommendations: Major stakeholders in the construction industry are therefore advised to use optimized GFRC as it is far cheaper and still possess the required strength needed for construction works.

Keywords: GFRC, Scheffe's (5,3) Polynomial Model, Optimization, Compressive strength ,Regression

1. INTRODUCTION

An optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. Every optimization problem requires an objective which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as workability, strength and durability. Scheffe's Polynomial Models are examples of optimization models. In this study, Scheffe's Third Degree Polynomial for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and glass fibre), simply stated as Scheffe's (5,3) polynomial model is developed.

In general, concrete is a very important material widely used in construction since ancient time. Concrete is of no doubt an important building material. According to Neville (1990), concrete plays a crucial part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. At the same time, it also has a major disadvantage which is that concrete is inherently a brittle material. Also, concrete is known for its problem associated with its low tensile strength compared to its compressive strength. As a result of this, many new technologies of concrete and some modern concrete specification approach were introduced. One of the technologies introduced for concrete was the addition of steel bars to reinforce its tension zone. This enables concrete gain an amount of tensile strength and thus reducing its brittle nature. Over the years the reinforcement (usually steel bars) has been replaced with other materials like glass fibre to further increase both its tensile strength and compressive strength and also, produce light weighted reinforced concrete unlike when reinforced with steel bars. Concrete's compressive strength is one of the most useful properties of concrete and in most structural applications, concrete primarily resists compressive stress.

Glass Fibre Reinforced Concrete (GFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced with a homogenous tiny strands of Alkaline Resistant (AR) glass fibre. Although GFRC has a similar density as concrete the product from it are 75% lighter due to the thin 10-15mm skin thickness used. For instance, a cladding panel manufactured from 100mm thick precast concrete would weigh 240kg/m^2 compared to a similar GFRC panel of 40-50kg/m^2 .GFRC can last as long as pre-cast concrete and can perform better when exposed to harsh or severe weather conditions due to absence of steel reinforcement that has the tendency to corrode. By using glass fibre as the matrix bound by the cementitious adhesion, substantial increase in the flexural strength and impact strength are achieved without losing the superb aging properties of the concrete. The combination of cement, fine aggregate and glass fibre allows the homogenously reinforced part (GFRC) to be made much thinner than one with only intermittent reinforcement.

The present study therefore presents a formulation of an optimization model that will optimize the strength of GFRC. It focuses on the use of scheffe's third degree polynomial model to optimize the strength of GFRC. In recent years, many researchers have used Scheffe's method to carry out one form of optimization project or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate

Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. Rao and others (2011) investigated the effect of size and shape of specimen on compressive strength of GFRC. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). That is to say, no work has been done on the use of Scheffe's method to optimize the compressive strength of GFRC except, the work by Nwachukwu and others (2017) which is based on Scheffe's Second Degree Polynomial. Henceforth, the need for this research work, whose results will be used to compare the work of Nwachukwu and others (2017). The Scheffe's theory is very relevant that it could predict the compressive strength of the GFRC concrete cubes if the mix ratios are known and vice versa

2. DEVELOPMENT OF THE OPTIMIZATION MODEL USING SCHEFFE'S THIRD DEGREE POLYNOMIAL

According to Aggarwal (2002), a simplex lattice is a structural representation of lines joining the atoms of a mixture, and these atoms are constituent components of the mixture. For GFRC mixture, the constituent elements are the water, cement, fine aggregate (sand), coarse aggregate and glass fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1\ ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

Where $X_i \geq 0$ and $i = 1, 2, 3 \dots q$, and q = the number of mixtures

2.1 THE SIMPLEX LATTICE DESIGN

The (q,m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1;\quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 3.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2 = 6$ points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$.

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \dots X_q$ is given in form of:

$$Y = b_0 + \sum bi\, xi + \sum bij\, xj + \sum bi\, jxjxk + + \sum bi^{j_2} + \dots i_n xi_2 xi_n \qquad \textbf{(2)}$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response which represents the property under study, which ,in this case is the compressive strength.

This research work was based on the (5, 3) simplex hence the actual form of Eqn. (2) will be developed for (5, 3) lattice subsequently.

2.2. RELATIONSHIP BETWEEN PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, the relationship between the pseudo components and the actual components is given as:

$$Z = A * X \quad \textbf{(3)}$$

Where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging the equation

$$X = A^{-1} * Z \quad \textbf{(4)}$$

In this research work a five component concrete mix constituents cement, river sand as fine aggregate, granite as coarse aggregate, water/cement (w/c) ratio and glass fibre were on focus .The space to use in the analysis will be (q – 1), which is equal to four dimensional factor spaces. A four dimensional factor space is an imaginary dimensional space (q, m) simplex lattice designs as shown in Fig.1 for (5, 3) simplex lattice design.

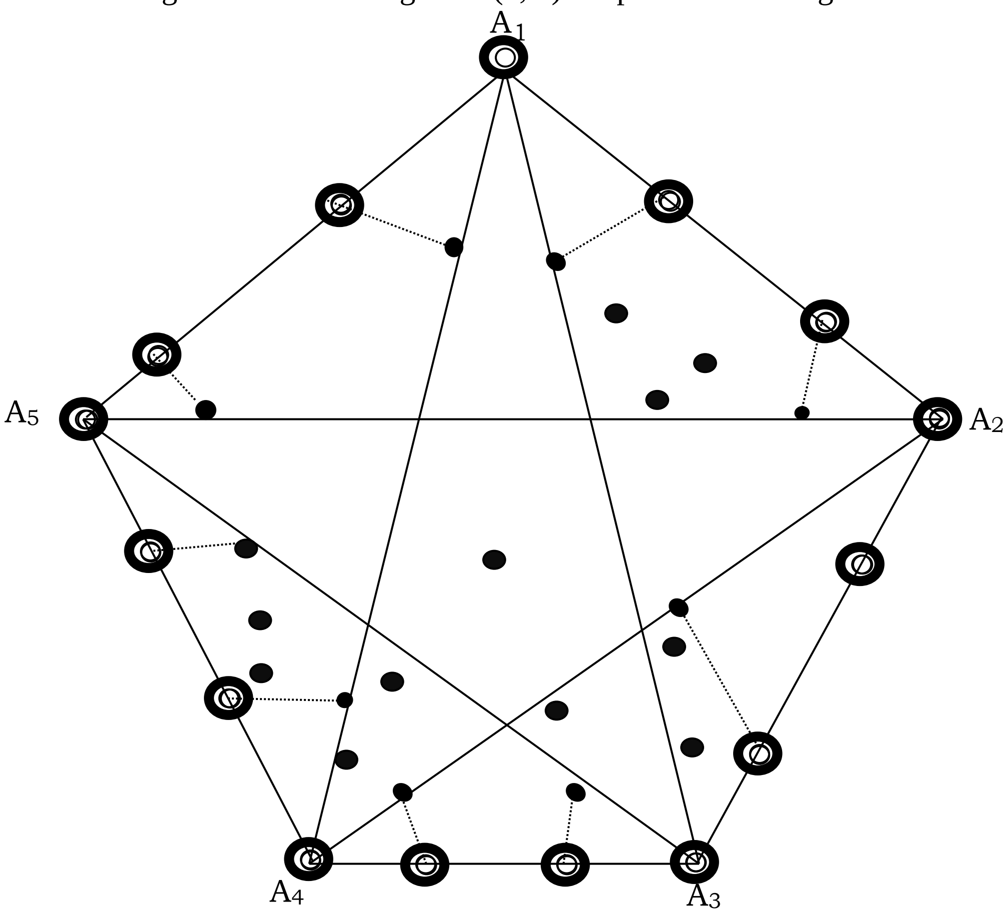

Fig 1: Imaginary Space Showing Four-Dimensional Factor Space

Let A_{jk} (X_{ijk}) designate arbitrary quantities of the five pseudo components of the mix at an arbitrary point on the factor space where A_{jk} is an arbitrary point on the factor space and X_{ijk} is the arbitrary quantities of all pseudo components at X_i at an arbitrary point, A_{jk}. In general X_{ijk} can take the form of:

$X_{ijk} = X_{1jk}, X_{2jk}, X_{3jk}, X_{4jk}, X_{5jk}$. **(5)**

The quantities of the five pseudo components at the 35 points are as follows: each X_i can take m+1 = 3 possible values i. e $X_i = 0, \frac{1}{2}, 1$. Then the possible design points, are:

A_1 (1,0,0,0,0); A_2 (0,1,0,0,0); A_3 (0,0,1,0,0); A_4 (0,0,0,1,0), A_5 (0,0,0,0,1); A_{112} (2/3. 1/3, 0, 0, 0,); A_{122} = (1/2, 2/3, 0,0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{133} (1/3, 0, 0, 2/3, 0, 0); A_{114} (2/3, 0,0,1/3,0); A_{114} (1/3, 0, 0, 2/3, 0); A_{115}, (2/3, 0, 0, 0, 1/3); A_{115} (1/3, 0,0,0, 2/3); A_{223} (0, 2/3, 1/3, 0,0); A_{223} (O, 1/3, 0,0); A_{224} (0,0 2/3, 0, 1/3, 0); A_{224} (0, 1/3, 0, 2/3,0); A_{225} (0, 2/3, 0,0, 1/3); A_{255} (0, 1/3, 0, 0, 2/3); A_{334} (0,0, 2/3, 1/3, 0); A_{344} (0,0,1/3, 2/3,0), A355 (0,0,2/3,0, 1/3); A_{355} (0,0,1/3,0, 2/3); A_{445} (0,0,0, 2/3, 1/3); A_{445} (0,0,0, 1/3, 2/3); A_{123} (1/3, 1/3, 1/3, 0,0); A_{124} (1/3, 1,3, 0, 1/3, 0); A_{125} (1/3, 1/3, 0,0, 1/3); A_{134} (134 (1/3, 0, 1/3, 1/3, 0); A_{135} (1/3, 0, 1/3, 0, 1/3); A_{145} (1/3, 0, 0,1/3,1/3); A_{234} (0,1/3, 1/3,1/3, 0); A_{235} (0,1/3, 1/3, 0, 1/3); A_{245} (0, 1/3, 0, 1/3, 1/3); A_{345} (0,0,1/3,1/3, 1/3). **(6)**

2.3. FORMULATION OF REGRESSION EQUATION FOR SCHEFFE'S (5, 3) LATTICE

The regression equation by Scheffe (1958), otherwise known as response is given as:

$f(x) = Y = b^{0+} \sum bi\ xj\ ^{+} \sum bi\ xj\ ^{+} \sum bij\ xi\ xj\ \ xk\ ^{+} \sum bi\ ^{i}{}_2 + \ldots i_n x i_2 x i_n$ **(7)**

Where $1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \ldots \le i_n \le q$ respectively

b_0 is the arbitrary constant and y is the response, and this response is a polynomial function of pseudo component of the mix.

Hence, for Scheffe's (5,3) simplex lattice, the regression equation is derived from Eqn.(7) and given as follows:

$Y = b_0 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1^2X_2 + b_{113}X_1^2X_3 + b_{114}X_1^2X_4 + b_{115}X_1^2X_5 + b_{22}X_2^2 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}X_2^2X_3 + b_{224}X_2^2X_4 + b_{225}X_2^2X_5 + b_{33}X_3^2 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3^2X_4 + b_{335}X_3^2X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4^2X_5 + b_{55}X_5^2 + b_{555}X_5^3$ **(8)**

Multiplying Eqn. (1) by b_0 yields Egn. (9)

$b_0 = b_0X_1 + b_0X_2 + b_0x_3 + b_0X_4 + b_0X_5$ **(9)**

By multiplying Eqn. (1) successively by X_1, X_2, X_3, X_4 and X_5 and re-arranging, we obtained Eqns. (10) – (14).

$X_1^2 = X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5$ **(10)**

$X_2^2 = X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5$ **(11)**

$X_3^2 = X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5$ **(12)**

$X_4^2 = X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5$ **(13)**

$$X_5^2 = X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5 \quad \textbf{(14)}$$

Substituting Eqns.(9 -14) into Eqn. (8) yields Eqn. (15)

$$Y = b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_0X_5 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11} (X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5) + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} (X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5)X_2 + b_{113}(X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5) X_3 + b_{114}(X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5) X_4 + b_{115} (X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5) X_5 +b_{22}(X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5) + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}(X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5)X_3 + b_{224}(X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5) X_4 + b_{225}(X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5) X_5 + b_{33}(X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5) + b_{34}X_3X_4 + b_{35}X_3X_5+ b_{333}X_3^3 + b_{334}(X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5) X_4 + b_{335}(X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5)X_5 + b_{44}(X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5) + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}(X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5) X_5 + b_{55}(X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5) + b_{555}X_5^3 \quad \textbf{(15)}$$

Expanding Eqn. (15) , we have

$$Y = b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_0X_5 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11} X_1 - b_{11}X_1X_2 - b_{11}X_1X_3 - b_{11}X_1X_4 - b_{11}X_1X_5 + b_{12}X_1X_2 + b_{13}X_1X_3+ b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1 X_2 - b_{112} X_1 X_2^2 - b_{112} X_1 X_2X_3 - b_{112} X_1 X_2X_4 - b_{112} X_1 X_2X_5 + b_{113}X_1 X_3 - b_{113}X_1X_2 X_3 - b_{113}X_1 X_3^2 - b_{113}X_1 X_3X_4 - b_{113}X_1 X_3X_5 + b_{114}X_1 X_4 - b_{114}X_1X_2 X_4 - b_{114}X_1X_3 X_4 - b_{114}X_1 X_4^2 - b_{114}X_1 X_4X_5 + b_{115} X_1 X_5 - b_{115}X_1X_2 X_5 - b_{115}X_1X_3 X_5 - b_{115}X_1X_4 X_5 - b_{115}X_1 X_5^2 + b_{22}X_2 - b_{22}X_1X_2 - b_{22}X_2X_3 - b_{22}X_2X_4 - b_{22}X_2X_5 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}X_2 X_3 - b_{223}X_1X_2 X_3 - b_{223}X_2 X_3^2 - b_{223}X_2 X_3X_4 - b_{223}X_2 X_3X_5 + b_{224} X_2 X_4 - b_{224} X_1X_2 X_4 - b_{224} X_2X_3 X_4 - b_{224} X_2 X_4^2 - b_{224} X_2 X_4X_5 + b_{225}X_2 X_5 - b_{225}X_1X_2 X_5 - b_{225}X_2X_3 X_5 - b_{225}X_2X_4 X_5 - b_{225}X_2 X_5^2 + b_{33}X_3 - b_{33}X_1X_3 - b_{33}X_2X_3 - b_{33}X_3X_4 - b_{33}X_3X_5 + b_{34}X_3X_4 + b_{35}X_3X_5+ b_{333}X_3^3 + b_{334}X_3 X_4 - b_{334}X_1X_3 X_4 - b_{334}X_2X_3 X_4 - b_{334}X_3 X_4^2 - b_{334}X_3 X_4X_5 + b_{335}X_3 X_5 - b_{335}X_1X_3 X_5 - b_{335}X_2X_3 X_5 - b_{335}X_3X_4 X_5 - b_{335}X_3 X_5^2 + b_{44}X_4 - b_{44}X_1X_4 - b_{44}X_2X_4 - b_{44}X_3X_4 - b_{44}X_4X_5 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4 X_5 - b_{445}X_1X_4 X_5 - b_{445}X_2X_4 X_5 - b_{445}X_3X_4 X_5 - b_{445}X_4 X_5^2 + b_{55} X_5 - b_{55} X_1X_5 - b_{55} X_2X_5 - b_{55} X_3X_5 - b_{55} X_4X_5 + b_{555}X_5^3 \quad \textbf{(16)}$$

Collecting like terms in Eqn. (16) yields Eqn. (17)

$$Y= X_1 [b_0 + b_1 + b_{11}] + X_2 [b_0 +b_2 + b_{22}] + X_3 [b_0+ b_3 + b_{33}] + X_4 [b_0 + b_4 + b_4] + X_5 [b_0 + b_5 + b_{55}] + X_1 X_2 [b_{12} - b_{11} - b_{22} + b_{112}] + X_1 X_3 [b_{13} - b_{11} - b_{33} + b_{113}] + X_1 X_4 [b_{14} - b_{11} - b_{44} + b_{114}] + X_1 X_5 [b_{15} - b_{11} - b_{55} + b_{115}] + X_1^3 [b_{111}] + X_1 X_2^2 [- b_{112}] + X_1 X_3^2 [- b_{113}] + X_1 X_4^2 [- b_{114}] + X_1 X_5^2 [- b_{115}] + X_1 X_2 X_3 [- b_{112} - b_{113} - b_{223}] + X_1 X_2 X_4 [- b_{112} - b_{114} - b_{224}] + X_1 X_2 X_5 [-b_{112} - b_{115} - b_{225}] + X_1 X_3 X_4 [- b_{113} - b_{114} - b_{334}] + X_1 X_3 X_5 [- b_{113} - b_{115} - b_{335}] + X_1 X_4 X_5 [- b_{114} - b_{115} - b_{445}] + X_2 X_3 [b_{23} - b_{22} - b_{33} + b_{223}] + X_2 X_4 [b_{24} - b_{22} - b_{44} + b_{224}] + X_2 X_5 [b_{25} - b_{22} - b_{55} + b_{225}] + X_2^2 [b_{222}] + X_2 X_3^2 [- b_{223}] + X_2 X_4^2 [- b_{224}] + X_2X_5^2 [- b_{225}] + X_2 X_3 X_4 [-b_{22} +b_{224} - b_{334}] + X_2 X_3 X_5 [- b_{223} - b_{225} - b_{335}] + X_2 X_4 X_5 [- b_{224} - b_{225} - b_{445}] + X_3 X_4 [b_{34} - b_{33} - b_{44} + b_{334}] + X_3 X_5 [b_{35} - b_{33} - b_{55} + b_{335}] + X_3^2 [b_{333}] + X_3 X_4^2 [-b_{334}] + X_3 X_5^2 [-b_{335}] + X_3 X_4 X_5 [- b_{334} - b_{335} - b_{445}] + X_4 X_5 [b_{45} - b_{44} - b_{55}+ b_{445}] + X_4^2 [b_{444}] + X_4 X_5^2 [- b_{445}] + X_5^2 [b_{555}] \quad \textbf{(17)}$$

Let

$[b_0 + b_1 + b_{11}] = \beta_1$; $[b_0 + b_2 + b_{22}] = \beta_2$; $[b_0 + b_3 + b_{33}] = \beta_3$; $[b_0 + b_4 + b_{44}] = \beta_4$; $[b_0 + b_5 + b_{55}] = \beta_5$; $[b_{12} - b_{11} - b_{22} + b_{112}] = \beta_{12}$.; $[b_{13} - b_{11} - b_{33} + b_{113}] = \beta_{13}$; $[b_{14} - b_{14} - b_{11} - b_{44} + b_{114} \} = \beta_{14}$; $[b_{15} - b_{11} - b_{55} + b_{115}] = \beta_{15}$; $[- b_{112}] = \gamma_{12}$; $[- b_{113}] = \gamma_{13}$; $[- b_{114}] = \gamma_{14}$; $[- b_{115}] = \gamma_{15}$; $[-b_{112}- b_{113} -b_{223}] = \beta_{123}$; $[-b_{112}- b_{114} - b_{224}] = \beta_{124}$; $[-b_{112}- b_{115} -b_{225}] = \beta_{125}$; $[- b_{113}- b_{114} -b_{334}] = \beta_{134}$; $[-b_{113}- b_{115} -b_{335}] = \beta_{135}$; $[-b_{113}- b_{115} -b_{445}] = \beta_{145}$; $[b_{23} - b_{22} - b_{33} + b_{223}$

] = β_{23}; [$b_{24} - b_{22} - b_{44} + b_{224}$] = β_{24}; [$b_{25} - b_{22} - b_{55} + b_{225}$] = β_{25}; [$- b_{223}$] = γ_{23}; [$- b_{224}$] = γ_{24}; [$- b_{225}$] = γ_{25}; [$-b_{223} - b_{224} - b_{334}$] = β_{234}; [$-b_{223} - b_{225} - b_{335}$] = β_{335}; [$-b_{224} - b_{225} - b_{445}$] = β_{245}; [$b_{34} - b_{33} - b_{44} + b_{334}$] = β_{34}; [$b_{35} - b_{33} - b_{55} + b_{335}$] = β_{35}; [$- b_{334}$] = γ_{34}; [$- b_{335}$] = γ_{35}; [$-b_{334} - b_{335} - b_{445}$] = β_{345}; [$b_{45} - b_{44} - b_{55} + b_{445}$] = β_{45}; [$- b_{445}$] = γ_{45}; **(18)**

Substituting Eqns.(18) into Eqn. (17) yields Eqn. (19)

$$Y= \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \gamma_{12}X_1X_2^2 + \gamma_{13}X_1X_3^2 + \gamma_{14}X_1X_4^2 + \gamma_{15}X_1X_5^2 + \beta_{123}X_1X_2X_3 + \beta_{124}X_1X_2X_4 + \beta_{125}X_1X_2X_5 + \beta_{134}X_1X_3X_4 + \beta_{135}X_1X_3X_5 + \beta_{145}X_1X_4X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \gamma_{23}X_2X_3^2 + \gamma_{24}X_2X_4^2 + \gamma_{25}X_2X_5^2 + \beta_{234}X_2X_3X_4 + \beta_{235}X_2X_3X_5 + \beta_{245}X_2X_4X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \gamma_{34}X_3X_4^2 + \gamma_{35}X_3X_5^2 + \beta_{345}X_3X_4X_5 + \beta_{45}X_4X_5 + \gamma_{45}X_4X_5^2 \quad \mathbf{(19)}$$

Equation (19) is the regression equation for Scheffe's (5, 3) simplex

2.4 .DETERMINATION OF THE COEFFICIENTS OF THE SCHEFFE'S (5, 3) POLYNOMIAL

Let Y_i = response function for the pure component, *i*

Then, through expansion of the work of Obam (2006), it can be established that:

$\beta_1 = Y_1$; $\beta_2=Y_2$; $\beta_3=Y_3$; $\beta_4= Y_4$; and $\beta_5= Y_5$ **20(a-e)**

$\beta_{12} = 9/4(Y_{112} + Y_{122} - Y_1 - Y_2)$; $\beta_{13} =9/4 (Y_{113}+ Y_{133}-Y_1-Y_3)$; $\beta_{14} = 9/4 (Y_{114}+Y_{144}-Y_1-Y_4)$ **21(a-c)**

$\beta_{15} = 9/4(Y_{115}+Y_{155}-Y_1-Y_5)$; $\beta_{23}=9/4 (Y_{223} +Y_{233}-Y_2-Y_3)$; $\beta_{24}=9/4 (Y_{224}+Y_{244}-Y_2-Y_4)$ **22(a-c)**

$\beta_{25} = 9/4(Y_{225}+Y_{255}-Y_2-Y_5)$; $\beta_{34}=9/4(Y_{334}+Y_{344}-Y_3-Y_4)$; $\beta_{35}=9/4(Y_{335}+Y_{355}-Y_3-Y_5)$ **23(a-c)**

$\beta_{45} = 9/4(Y_{445}+Y_{455}-Y_4-Y_5)$; $\gamma_{12} = 9/4(3Y_{112}+3Y_{122}-Y_1+Y_2)$; $\gamma_{13}=9/4(3Y_{113}+3Y_{133}-Y_1+Y_3)$ **24(a-c)**

$\gamma_{14} = 9/4(3Y_{114}+3Y_{144}-Y_1+Y_4)$; $\gamma_{15} =9/4(3Y_{115}+3Y_{115}-Y_1+Y_5)$; $\gamma_{23}=9/4(3Y_{223}+3Y_{233}-Y_2+Y_3)$ **25(a-c)**

$\gamma_{24} = 9/4 (3 Y_{224}+3 Y_{244}-Y_2+Y_4)$; $\gamma_{25}=9/4(3Y_{225}+3Y_{255}-Y_2+Y_5)$; $\gamma_{34} = 9/4(3Y_{334}+3Y_{344}Y_3 +Y_4)$ **26(a-c)**

$\gamma_{35} =9/4(3Y_{335}+3Y_{355}-Y_3+Y_5)$; $\gamma_{45}=9/4(3Y_{445}+3Y_{455}-Y_4+Y_5)$ **27(a-b)**

$\beta_{123} = 27Y_{123} -27/4(Y_{112}+Y_{122}+Y_{113}+_{133}+Y_{223}+Y_{233}) + 9/4(Y_1+Y_2+Y_3)$ **(28)**

$\beta_{124} =27Y_{124} -27/4(Y_{112}+Y_{122}+Y_{114}+Y_{144}+Y_{224}+Y_{244}) +9/4(Y_1+Y_2+Y_4)$ **(29)**

$\beta_{125} =27Y_{125}-27/4(Y_{112}+Y_{122}+Y_{115}+Y_{155}+Y_{225}+Y_{255}) + 9/4(Y_1+Y_2+Y_5)$ **(30)**

$\beta_{134}=27Y_{134}-27/4(Y_{113}+Y_{133}+Y_{114}+Y_{144}+Y_{334}+Y_{344}) + 9/4(Y_1+Y_3+Y_4)$ **(31)**

$\beta_{135} =27Y_{135} -27/4(Y_{113}+Y_{133}+Y_{115}+Y_{155}+Y_{335}+Y_{355}) + 9/4(Y_1+Y_3+Y_5)$ **(32)**

$\beta_{145} = 27Y_{145} - 27/4(Y_{114}+Y_{144} +Y_{115} + Y_{155}+Y_{445}+Y_{455}) + 9/4(Y_1+Y_4 +Y_5)$ **(33)**

$\beta_{234} = 27Y_{234}- 27 /4(Y_{223}+Y_{233}+Y_{224}+Y_{244}+Y_{334} + Y_{344}) +9/4(Y_2+Y_3+Y_4)$ **(34)**

$\beta_{235} = 27Y_{235} -27/4(Y_{223}+Y_{233}+Y_{225}+Y_{255}+Y_{335}+Y_{355}) + 9/4(Y_2+Y_3+Y_5)$ **(35)**

$\beta_{245}=27Y_{245}-27/4(Y_{224}+Y_{244}+Y_{225}+Y_{255}+Y_{445}+Y_{455}) + 9/4(Y_2+Y_4+Y_5)$ **(36)**

$\beta_{345}=27Y_{345}-27/4(Y_{334}+Y_{344}+ Y_{335}+ Y_{355} + Y_{445} + Y_{455}) +9/4(y_1+y_4+y_5)$ **(37)**

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2.5. MIX RATIO

Since the simplex operates in such a way that sum of all components must be one, it becomes impossible to use normal mix ratios such as 1:2:4 or 1:3:6. Rather transformation was carried out from actual to pseudo components. .

2.5.1. ACTUAL MIX RATIO

Based on past experience and literature, the arbitrary mix proportions prescribed for the vertices of the pentagon is shown in Fig.2.It is in the order of (W/C:C:F.A:C.A:G.F) which represents water/ cement ratio, cement, fine aggregate, coarse aggregate and glass fibre respectively

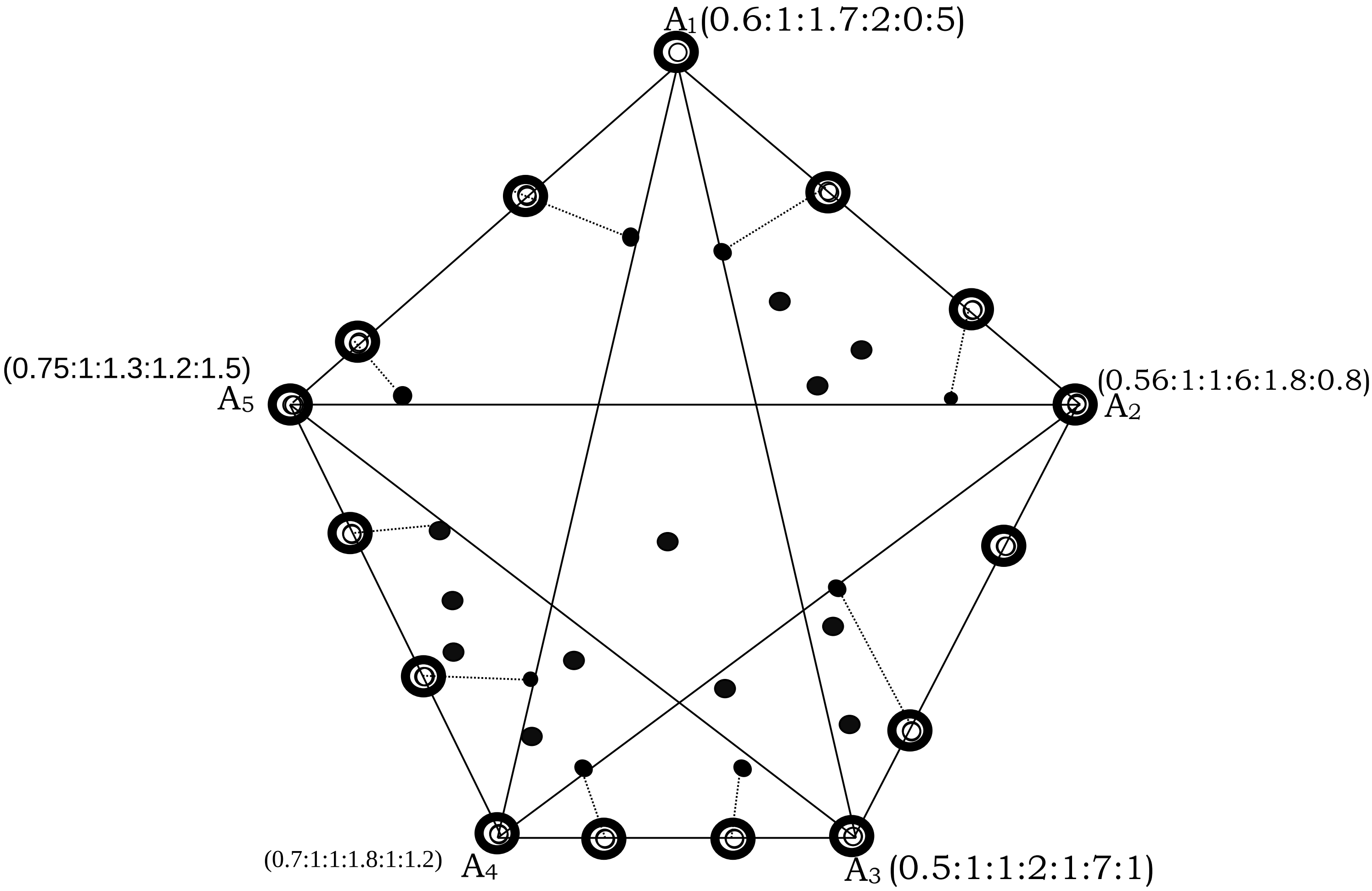

Fig. 2: Vertices of a (5, 3) lattice (actual)

2.5.2: PSEUDO MIX RATIO

From Eqn.(6), the corresponding pseudo mix ratios are shown in Fig. 3

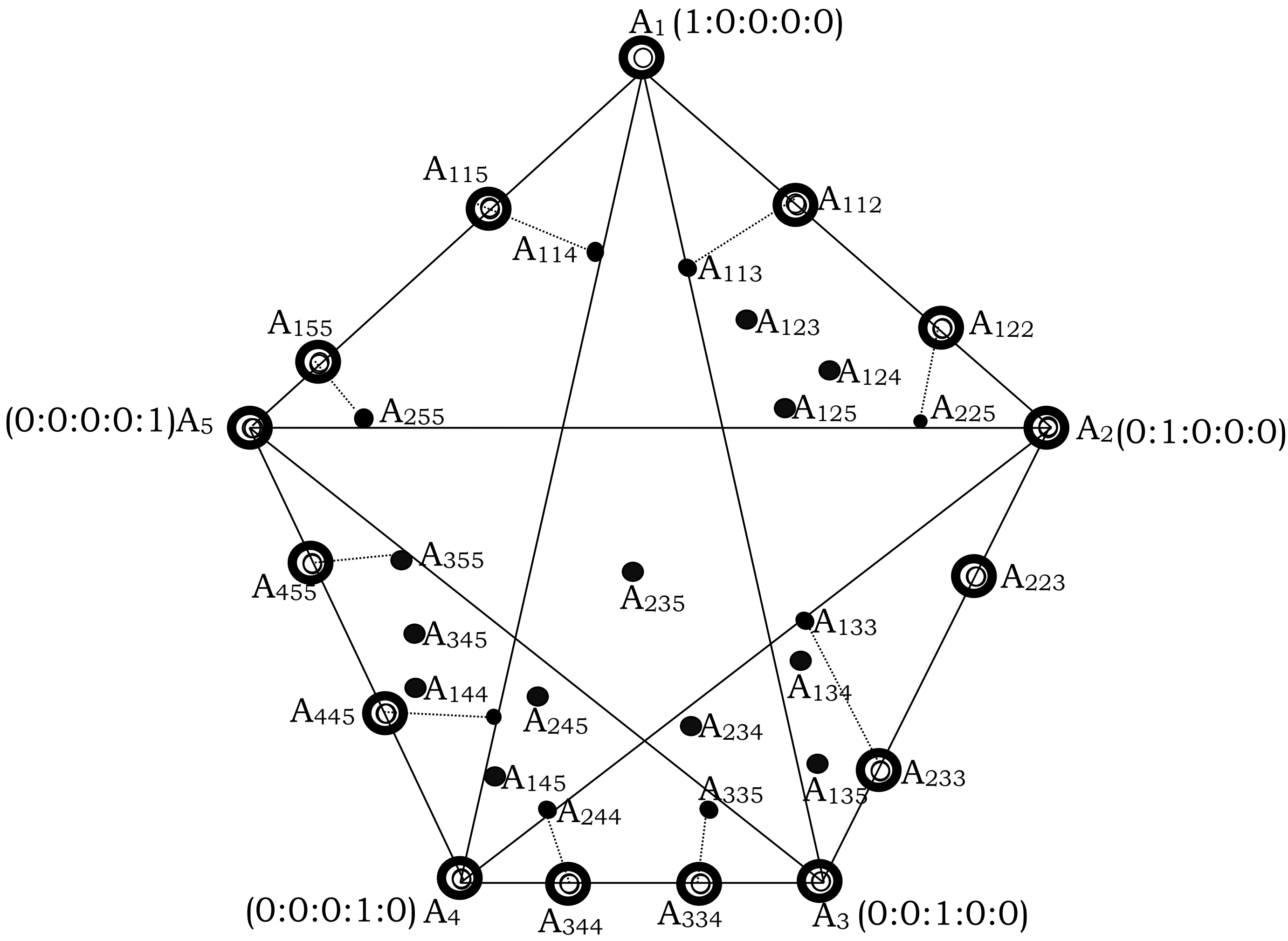

Fig. 3 vertices of a (5, 3) lattice (pseudo)

2.5.3. COMPONENTS TRANSFORMATION

If X represent the pseudo component and Z the actual components. For component transformation, Sheffes suggested the following equations:

$$X = B * Z; \qquad Z = A * X \qquad \textbf{38 (a-b)}$$

Where A = matrix whose elements are arbitrary mix proportions; B = the inverse of matrix A; Z = matrix of the actual components and X = matrix of the pseudo component obtained from Fig. (3).

Considering a (5, 3) simplex lattice, it follows that there are five components in the mixture. Hence, Eqn. (38b) becomes:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} \\ A_{122} & A_{222} & A_{223} & A_{242} & A_{225} \\ A_{331} & A_{332} & A_{223} & A_{224} & A_{225} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \qquad \textbf{(39)}$$

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Substituting the mix ratios from point A (Figures 2 and 3) respectively into Eqn. (39), we obtain:

$$\begin{pmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \textbf{(40)}$$

Transforming the R.H.S matrix and solving, we obtain

A_{111}= 0.67; A_{221}= 1; A_{331}= 1.7; A_{441}= 2; A_{551}= 0.5

The same approach is used to obtain the remaining values as shown in Eqn. (41)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \quad \textbf{(41)}$$

Considering mix ratios at the mid points from Eqn.(6) and substituting these pseudo mix ratios in turn into Eqn.(41) will yield the corresponding actual mix ratios.

For point A_{112}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \end{Bmatrix} \quad \textbf{(42)}$$

Solving, Z_1 = 0.63; Z_2 = 1.00; Z_3 = 1.67' Z_4 = 1.90; Z_5 = 1.60

The same approach goes for the remaining mid-point mix ratios

Hence, to generate the regression coefficients, 35 experimental tests were carried out and the corresponding mix ratios depicted in Table 1

Table 1: Mix Ratio for the (5.3) Lattice

Point	Pseudo		Component			Response	Actual		Component		
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	1	0	0	0	0	Y_1	0.67	1.00	1.70	2.0	0.5
2	0	1	0	0	0	Y_2	0.56	1.00	1.60	1.8	0.8
3	0	0	1	0	0	Y_3	0.50	1.00	1.20	1.7	1.0
4	0	0	0	0	0	Y_4	0.70	1.00	1.00	1.8	1.2
5	0	0	0	0	1	Y_5	0.75	1.00	1.30	1.2	1.5
112	0.67	0.33	0	1	0	Y_{112}	0.63	1.00	1.67	1.9	1.6
122	0.33	0.67	0	0	0	Y_{122}	0.60	1.00	1.63	1.8	0.7
113	0.67	0	0.33	0	0	Y_{113}	0.61	1.00	1.54	1.9	0.6
133	0.33	0	0.67	0	0	Y_{113}	0.56	1.00	1.37	1.8	0.8
114	0.67	0	0	0.33	0	Y_{114}	0.68	1.00	1.47	1.9	0.7
144	0.33	0	0	0.67	0	Y_{144}	0.69	1.00	1.23	1.8	0.9
115	0.67	0	0	0	0.33	Y_{115}	0.70	1.00	1.57	1.7	0.8
155	0.33	0	0	0	0.67	Y_{115}	0.72	1.00	1.43	1.4	1.1
223	0	0.67	0.33	0	0	Y_{223}	0.55	1.00	1.40	1.7	0.8
233	0	0.33	0.67	0	0	Y_{233}	0.52	1.00	1.20	1.7	0.9
224	0	0.67	0	0.33	0	Y_{224}	0.61	1.00	1.67	1.8	0.9
244	0	0.33	0	0.67	0	Y_{244}	0.66	1.00	1.73	1.8	1.0
225	0	0.67	0	0	0.33	Y_{225}	0.63	1.00	1.50	1.6	0.7
255	0	0.33	0	0	0.67	Y_{255}	0.69	1.00	1.40	1.4	0.6
234	0	0	0.67	0.33	0	Y_{334}	0.57	1.00	1.13	1.7	1.0
344	0	0	0.33	0.67	0	Y_{344}	0.64	1.00	1.07	1.7	1.1
335	0	0	0.67	0	0.33	Y_{355}	0.58	1.00	1.23	1.5	1.1
355	0	0	0.33	0	0.67	Y_{335}	0.67	1.00	1.27	1.3	1.3
445	0	0	0	0	0.67	Y_{445}	0.72	1.00	1.10	1.6	1.3
455	0	0	0	0.67	0.33	Y_{445}	0.73	1.00	1.20	1.4	1.4
123	0.33	0.33	0.33	0	0	Y_{123}	0.57	1.00	1.49	1.8	0.7
124	0.33	0.33	0	0.33	0	Y_{124}	0.64	1.00	1.09	1.8	0.8
125	0.33	0.33	0	0	0.33	Y_{125}	0.66	1.00	1.52	1.6	0.9
134	0.33	0.33	0.33	0.33	0	Y_{134}	0.62	1.00	1.29	1.8	0.8
135	0.33	0	0.33	0.33	0.33	Y_{135}	0.63	1.00	1.39	1.6	0.9
145	0.33	0	0	0	0.33	Y_{145}	0.70	1.00	1.32	1.6	1.0
234	0	0	0.33	0.33	0	Y_{234}	0.58	1.00	1.25	1.7	0.9
235	0	0.33	0.33	0	0.33	Y_{235}	0.60	1.00	1.32	1.5	1.0
245	0	0.33	0	0.33	0.33	Y_{245}	0.67	1.00	1.29	1.5	1.1
345	0	0	0.33	0.33	0.33	Y_{345}	0.64	1.00	1.6	1.5	1.2

2.5.4. CONTROL POINTS

Thirty five (35) different controls were predicted which according to Scheffe's (1958), their summation should not be greater than one. They are:

C_1 (0.25: 0.25: 0.25:0.25: 0); C_2 (0.25:0.25: 0.25:0: 0.25); C_3 (0.25:0.25:0.25:0.25); C_4 (0.25:0.0.25: 0.25:0.25); C_5 (0.0.25:0.25:0.25:0.25); C_{112} (0.2:0.2:0.2: 0.2:0.25); C_{122} (0.3:

0.3: 0.3: 0:0.1); C_{113} (0.3: 0.3: 0.3: 0:0.1); C_{133} (0.3:0.3:0.3:0.1); C_{114} (0.3:0: 0.3:0.3:0.1);C_{144} (0:0.3:0.3:0.3:0.1); C_{115}(0.1:0.3:0.3:0.3:0); C_{155}(0.3:0.1:0.3:0.3:0); C_{223}(0.3:0.3:0.1:0.3:0); C_{233} (0.1:0.2: 0.3:04:0); C_{224} (0.30:0.20:0:0 0.4:0); C_{244} (0.20: 0.20:010:0.10:0.40); C_{225} (0.30:0.10:0.30:0.20:0.10); C_{255} (0.25: 0.25:0.10:0.15:0.20); C_{334} (0.30: 0.30: 0.20:0.20: 0.10);C_{344} (0.10: 0.30: 0.30: 0.30:0); C_{335} (0.30: 0.30:0.20: 0.20: 0.20); C_{355} (0.25:0.15: 0.20: 0.20:0.20); C_{445}(0.10:0.20:0.30:0.40:0); C_{455}(0:0.40: 0.20: 0.30:0.10); C_{123} (0.25:0.10:0.40:0:0.25);C_{124} (0.30:0.20: 0.40:0.10,0); C_{135} (0.25:0.20:0.20:0.20:0.15); C_{134} (0.10:0.30:0 0.30:0.30); C_{135} (0.25:0.20: 0.20: 0.20: 0.15); C_{145} (0.10: 0.10: 0.10 : 0.30 : 0.40); C_{234} (0.40: 0.20: 0.10: 0.10: 0.30); C_{235} (0.25: 0.25: 0.15: 0.25: 0.10); C_{245} (0.15: 0.20: 0.10:0.25:0.30);C_{345} (0.30: 0.10: 0.20: 0.25: 0.15); **(43)**

Substituting these values into Eqn (41), gives the actual mixes values as follows:

For control point C_1;

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 075 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.61 \\ 1.00 \\ 1.38 \\ 1.83 \\ 0.5 \end{pmatrix} \quad \textbf{(44)}$$

The rest are shown in Table 2

Table 2: Actual and Pseudo Component of Scheffe (5,3) Lattice for Control Points

Point	Pseudo		Component			Control	Actual		Component		
	X_1	X_2	X_3	X_4	X_5	Point	Z_1	Z_2	Z_3	Z_4	Z_5
1	0.25	0.25	0.25	0.25	0	C_1	0.61	1	1.38	1.83	0.5
2	0.25	0.25	0.25	0	0.25	C_2	0.62	1	1.45	1.68	0.8
3	0.25	0.25	0	0.25	0.25	C_3	0.67	1	1.40	1.70	1
4	0.25	0	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.2
5	0	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.5
12	0.2	0.2	0.2	0.2	0.2	C_{112}	0.64	1	1.36	1.70	0.65
22	0.3	0.3	0.3	0.1	0	C_{122}	0.59	1	1.45	1.83	0.75
13	0.3	0.3	0.3	0	0.1	C_{113}	0.59	1	1.48	1.77	0.85
33	0.3	0.3	0	0.3	0.1	C_{133}	0.65	1	1.42	1.80	1
14	0.3	0	0.3	0.3	0.1	C_{114}	0.64	1	1.30	1.77	0.9
44	0	0.3	0.3	0.3	0.1	C_{144}	0.60	1	1.27	1.71	1
115	0.1	0.3	0.3	0.3	0	C_{155}	0.60	1	1.31	1.79	1.55
115	0.3	0.1	0.3	0.3	0	C_{155}	0.62	1	1.33	1.83	1.1
223	0.3	0.1	0.3	0.3	0	C_{223}	0.63	1	1.41	1.85	1.25
233	0.1	0.2	0.3	0.4	0	C_{233}	0.61	1	1.25	1.79	1.35
224	0.30	0.20	0.10	0.4	0	C_{224}	0.64	1	1.35	1.85	0.89
244	0.20	0.20	0.10	0.10	0.40	C_{244}	1.40	1	1.04	1.59	1.08
225	0.30	0.10	0.30	0.20	0.10	C_{225}	0.62	1	1.36	1.77	0.92
255	0.25	0.25	0.10	0.15	0.20	C_{255}	0.61	1	1.51	3.16	0.91
334	0.30	0.30	0.20	0.20	0.10	C_{334}	0.68	1	1.56	1.96	0.98

344	0.10	0.30	0.30	0.30	0	C_{344}	1.30	1	1.31	1.79	0.95
335	0.30	0.15	0.20	0.20	0.20	C335	0.65	1	0.96	1.05	0.97
355	0.25	0.20	0.20	0.20	0.20	C_{355}	0.64	1	1.37	1.71	0.79
445	0.10	0.20	0.30	0.40	0	C_{445}	0.61	1	1.25	1.79	0.99
455	0	0.10	0.20	0.30	0.10	C_{455}	0.61	1	1.31	1.72	1.03
123	0.25	0.10	0.40	0	0.25	C_{123}	0.61	1	1.39	1.66	0.98
124	0.30	0.20	0.40	0.10	0	C_{124}	0.58	1	1.41	1.82	0.83
125	0.15	0.15	0.20	0.10	0.40	C_{125}	0.65	1	1.36	1.57	1.11
134	0.10	0.30	0	0.30	0.30	C_{134}	0.67	1	1.34	1.65	1.10
135	0.25	0.20	0.20	0.20	0.15	C_{135}	0.74	1	1.38	2.08	0.88
145	0.10	0.10	0.10	0.30	0.40	C_{145}	0.68	1	1.27	1.57	1.19
234	0.40	0.20	0.10	0.10	0.30	C_{234}	0.73	1	1.61	1.87	1.03
235	0.25	0.25	0.15	0.25	0.10	C_{235}	0.63	1	1.39	1.78	0.93
245	0.15	0.20	0.10	0.25	0.30	C_{245}	0.66	1	1.34	1.64	1.09
345	0.30	0.10	0.20	0.25	0.15	C_{345}	0.64	1	1.34	1.75	0.96
15	0.3	0.3	0	0.3	0.1	C_{15}	0.65	1	1.42	1.80	1
23	0.3	0	0.3	0.3	0.1	C_{23}	0.64	1	1.30	1.77	0.9
24	0	0.3	0.3	0.3	0.1	C_{24}	0.60	1	1.27	1.71	1
25	0.1	0.3	0.3	0.3	0	C_{25}	0.60	1	1.31	1.79	1.15
34	0.3	0.1	0.3	0.3	0	C_{34}	0.62	1	1.33	1.83	1.1
35	0.3	0.3	0.1	0.3	0	C_{35}	0.63	1	1.41	1.85	1.25
45	0.1	0.2	0.3	0.4	0	C_{45}	0.61	1	1.25	1.79	1.35

The actual component as transformed from Eqn. (41) and Table (1) and (2) were used to measure out the quantities of water (Z_1), cement (Z_2), fine aggregate as sand (Z_3), coarse aggregate (Z_4) and glass fibre (Z_5) in their respective ratios for the concrete cube strength test.

3. MATERIALS AND METHODS

3.1 MATERIALS

The materials investigated are the mixture of cement, water, fine and coarse aggregate and glass fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size downgraded to 4.75mm obtained from a local stone market was used in the experimental investigation .Also; Glass Fibre of alkali resistant type (CEM-FIL) was used in the experimental investigation and water drawn from the clean water source.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 70 mix ratios were to be used to produce 70 prototype concrete cube. Fifteen (35) out of the 70 mix ratios were as control mix ratios to produce 35 cubes for the conformation of the adequacy of the mixture design given by the Eqn. (19). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Testing was conducted immediately after the specimen was removed from the curing process and dried. Smooth surface metal plate (serving as base plate) was placed at the bottom and top of each of the specimen cube so as to ensure uniform distribution of load for accurate crushing. Two samples were crushed for each mix ratio. The compressive strength was then calculated using the formula below:

$$\text{Compressive Strength} = \frac{\text{Average failure Load (N)}}{\text{Cross- sectional Area (mm}^2)} \quad \frac{P}{A} \qquad \textbf{(45)}$$

4. RESULTS AND DISCUSSION

4.1. COMPRESSIVE STRENGTH AND BULK DENSITY TEST RESULTS

The results of the compressive strength ($R_{esponse}$, Y_i) based on a 28-days strength is presented in Table 3. These were calculated from Eqn.(45)

Table 3: 28th Day Compressive Strength Values and their Corresponding Densities for the Initial Experimental Tests.

Response Symbol	Replicate	Average weight (KN)	Volume (M^3)	Average bulk density	Crushing load (KN)	Cross sectional area (MM^2)	Strength (Nmm^2)	Average strength (Nmm^2)
Y_1	A				342		15.20	
	B	8.50	0.003375	2519	390	22500	17.33	16.89
	C				408		18.13	
Y_2	A				384		17.07	
	B	8.34	0.003375	2471	348	22500	15.47	16.03
	C				350		15.56	
Y_3	A				300		13.33	
	B	8.13	0.003375	2409	380	22500	16.89	15.26
	C				350		15.56	
Y_4	A				478		21.25	
	B	8.54	0.003375	2542	461	22500	20.47	19.91
	C				405		18.00	
Y_5	A				486		2160	
	B	8.51	0.003375	2533	480	22500	21.34	19.65
	C				360		16.00	
Y_{112}	A				380		16.89	
	B	8.25	0.003375	2444	360	22500	16.00	16.00
	B				340		15.11	
Y_{122}	A				300		13.33	
	B	8.25	0.003375	2444	350	22500	15.56	14.37
	C				320		14.22	

Y122	A				342		15.20	
	B	8.08	0.003375	2415	348	22500	15.47	15.20
	C				336		14.93	
Y133	A				320		14.22	
	B	8.22	0.003375	2436	330	22500	14.67	15.26
	C				380		16.89	
Y114	A				350		15.56	
	B	8.30	0.003375	2436	300	22500	13.33	14.00
	C				295		13.11	
Y144	A				300		13.32	
	B	8.24	0.003375	2444	348	22500	15.47	14.84
	C				354		15.73	
Y115	A				384		17.06	16.80
	B	8.22	0.003375	2436	390	22500	17.33	
	C				360		16.00	
Y155	A				360		16.00	
	B	8.64	0.003375	2560	466	22500	20.78	18.06
	C				432		19.20	
Y223	A				312		13.87	
	B	8.10	0.003375	2400	360	22500	16.00	14.40
	C				300		13.33	
Y233	A				336		14.93	
	B	8.32	0.003375	2465	390	22500	17.34	16.09
	C				360		16.00	
Y224	A				340		15.11	16.59
	B	8.30	0.003375	2459	380	22500	16.89	
	C				400		17.78	
Y244	A				350		15.56	
	B	8.34	0.003375	2471	345	22500	15.33	14.74
	C				300		13.33	
Y225	A				360		15.56	
	B	8.22	0.003375	2436	385	22500	15.33	16.67
	C				380		13.33	
Y255	A				485		21.55	
	B	8.50	0.003375	2518	495	22500	22.00	21.82
	C				493		21.91	
Y334	A				490		21.78	
	B	8.48	0.003375	2512	470	22500	20.89	19.63
	C				365		16.22	
Y344	A				380		16.89	
	B	8.36	0.003375	2477	365	22500	16.22	16.15
	C				345		15.33	
Y335	A				345		15.32	15.26
	B	8.10	0.003375	2400	350	22500	15.36	
	C				335		14.89	

Y_{355}	A B C	8.33	0.003375	2221	300 348 350	22500	13.32 15.47 15.56	14.78
Y_{445}	A B C	8.25	0.003375	2444	385 370 360	22500	17.11 16.44 16.	16.52
Y_{455}	A B C	8.28	0.003375	2453	380 400 375	22500	16.89 17.78 16.67	17.11
Y_{123}	A B C	8.35	0.003375	2474	390 385 395	22500	17.33 17.11 17.56	17.33
Y_{124}	A B C	8.52	0.003375	2524	390 420 400	22500	17.33 18.67 17.78	17.93
Y_{125}	A B C	8.49	0.003375	2515	370 385 390	22500	16.44 17.11 17.33	16.96
Y_{134}	A B C	8.40	0.003375	2488	365 380 375	22500	16.22 16.89 16.67	16.59
Y_{135}	A B C	8.43	0.003375	2498	368 390 380	22500	16.36 17.33 17.51	16.86
Y_{145}	A B C	8.47	0.003375	2509	400 382 394	22500	17.78 16.98 17.11	17.43
Y_{234}	A B C	8.53	0.003375	2527	420 382 385	22500	18.67 16.98 17.11	17.59
Y_{235}	A B C	8.56	0.003375	2536	410 390 395	22500	18.22 17.33 17.56	17.70
Y_{245}	A B C	8.58	0.003375	2542	400 420 395	22500	17.78 18.67 17.56	18.00
Y_{345}	A B C	8.62	0.003375	2554	410 400 425	22500	18.22 17.78 18.89	18.29

4.2 EXPERIMENTAL (CONTROL) TEST RESULT

Table 4 shows the 28th day Compressive strength values and their corresponding density for the Control Test Experimental tests.

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Table 4: 28TH Day Compressive Strength Value and their corresponding Density for the Control Test Experimental Tests.

Response Symbol	Replicate	Average weight (KN)	Volume (M^3)	Average bulk density	Crushing load (KN)	Cross sectional area (MM^2)	Strength (Nmm^2)	Average strength (Nmm^2)
C_1	A B C	8.79	0.003375	26.04	364 465 419	22500	19.18	18.48
C_2	A B C	8.15	0.003375	2415	428 387 385	22500	18.00 17.20 16.1	17.10
C_3	A B C	8.45	0.003375	2504	338 423 388	22500	18.02 15.81 17.23	17.02
C_4	A B C	8.19	0.003375	2427	549 460 378	22500	19.41 20.46 21.81	20.56
C_5	A B C	8.01	0.003375	2373	457 412 506	22500	26.25 21.30 19.50	20.35
C_{112}	A B C	8.25	0.003375	2444	407 385 431	22500	18.07 17.12 19.17	18.12
C_{122}	A B C	7.89	0.003375	2338	348 408 414	22500	15.48 18.13 18.38	17.33
C_{113}	A B C	8.19	0.003375	2427	342 403 362	22500	15.20 17.90 16.10	16.40
C_{133}	A B C	8.07	0.003375	2391	397 347 375	22500	1565 17.60 16.45	16.56
C_{114}	A B C	8.25	0.003375	2444	389	22500	17.30 14.00 14.20	15.20
C_{144}	A B C	8.01	0.003375	2373	397 347 375	22500	15.65 16.45 17.65	16.56
C_{115}	A B C	8.07	0.003375	2391	410 401 407	22500	18.22 18.80 17.10	18.04

C_{155}	A	8.49	0.003375	2516	475	22500	21.10	19.56
	B				437		19.44	
	C				408		18.13	
C_{223}	A	7.97	0.003375	2361	349	22500	16.50	15.52
	B				399		14.73	
	C				300		15.33	
C_{233}	A	8.01	0.003375	2373	437	22500	19.40	17.30
	B				367		16.30	
	C				365		16.20	
C_{224}	A	8.02	0.003375	2376	458	22500	19.36	18.23
	B				365		17.20	
	C				408		18.13	
C_{244}	A	8.23	0.003375	2439	338	22500	16.50	15.65
	B				403		15.40	
	C				316		15.05	
C_{225}	A	7.98	0.003375	2364	360	22500	18.00	17.75
	B				386		19.15	
	C				452		16.10	
C_{255}	A	8.15	0.003375	2415	371	22500	15.50	16.39
	B				318		15.13	
	C				417		11.54	
C_{334}	A	8.21	0.003375	2433	441	22500	19.60	17.40
	B				320		14.20	
	C				4141		18.40	
C_{344}	A	799	0.003375	2367	360	22500	18.03	17.95
	B				404		17.93	
	C				448		17.90	
C_{344}	A	7.99	0.003375	2367	360	22500	18.03	17.95
	B				404		17.93	
	C				448		17.90	
C_{335}	A	7.89	0.003375	2338	420	22500	18.65	16.45
	B				344		15.30	
	C				347		15.40	
C_{355}	A	8.14	0.003375	2412	339	22500	15.07	15.88
	B				388		17.23	
	C				423		15.23	
C_{445}	A	8.29	0.003375	2456	383	22500	17.03	18.02
	B				448		19.93	
	C				385		17.10	
C_{455}	A	8.18	0.003375	2424	446	22500	19.81	18.56
	B				419		18.60	
	C				389		17.27	

C_{123}	A	8.28	0.003375	2453	441	22500	19.60	18.60
	B				367		16.30	
	C				448		19.90	
C_{124}	A	8.00	0.003375	2370	385	22500	17.10	18.75
	B				410		18.20	
	C				471		20.95	
C_{125}	A	8.16	0.003375	2418	363	22500	16.12	17.86
	B				410		18.20	
	C				433		19.26	
C_{134}	A	8.39	0.003375	2486	423	22500	16.25	17.54
	B				396		18.79	
	C				366		17.58	
C_{135}	A	8.40	0.003375	2489	351	22500	15.62	17.70
	B				423		18.70	
	C				421		18.78	
C_{145}	A	8.41	0.003375	2492	471	22500	19.55	18.35
	B				385		18.10	
	C				394		17.40	
C_{234}	A	8.08	0.003375	2394	468	22500	20.80	18.55
	B				392		17.40	
	C				393		17.45	
C_{235}	A	8.10	0.003375	2400	383	22500	17.00	18.90
	B				423		18.80	
	C				468		20.90	
C_{245}	A	8.13	0.003375	2409	458	22500	20.35	19.25
	B				458		20.25	
	C				386		17.15	
C_{345}	A	8.43	0.003375	2498	385	22500	18.12	19.85
	B				477		20.18	
	C				478		21.25	

4.3 REGRESSION EQUATION FOR COMPRESSIVE STRENGTH

From Eqns. (20) through (37) and Table 3, the coefficients of the Scheffe's third degree polynomial were determined as follows:

$\beta_1 = 16.89$; $\beta_2 = 14.93$; $\beta_3 = 16.80$; $\beta_4 = 19.91$; $\beta_5 = 19.65$;

$\beta_{12} = -3.74$; $\beta_{13} = -13.65$; $\beta_{14} = -3.11$; $\beta_{15} = -7.33$; $\beta_{23} = -1.8$; $\beta_{24} = -10.37$; $\beta_{25} = 3.11$; $\beta_{34} = 1.37$; $\beta_{35} = -10.96$; $\beta_{45} = -13.34$; $\gamma_{12} = 90.25$; $\gamma_{13} = 201.94$; $\gamma_{14} = 198.2$; $\gamma_{15} = 90.25$; $\gamma_{23} = 2\text{-}4.07$; $\gamma_{24} = 220.21$; $\gamma_{25} = 258.3$; $\gamma_{34} = 2.51.98$; $\gamma_{35} = 212.65$; $\gamma_{45} = 226.42$; $\beta_{123} = -40$; $\beta_{124} = -5.79$; $\beta_{125} = 131.84$; $\beta_{134} = -35.60$; $\beta_{135} = -75.95$; $\beta_{145} = -61.02$; $\beta_{234} = -69$; $\beta_{235} = -66.21$; $\beta_{245} = -82.53$; $\beta_{345} = -55.04$. **(46)**

Substituting the values of these coefficients in Eqn.(46) into Eqn. (19), yields the mathematical model for the optimization of the compressive strength of the concrete cubes made using glass fibre (GFRC) based on Scheffe's (5,3) polynomial.

4.4 VALIDATION AND TEST OF ADEQUACY OF THE MODEL

The model was analyzed statistically using Fisher test and the adequacy of the model was tested against the experimental results of the control points. The predicted values ($Y_{(predicted)}$) for the test control points were obtained by substituting the values of X_i into the Scheffe's (5,3) Polynomial Model Equation i.e. Revised Eqn. (19). These values were compared with the experimental result ($Y_{(observed)}$) given in Table 3 and there is no significant difference between the model experimental result and the theoretical expected result. Thus, the model is adequate.

4.5. COMPARISON BETWEEN SCHEFFE'S SECOND DEGREE (5,2) AND THIRD DEGREE (5,3)

POLYNOMIAL MODELS

The table of comparison is shown in Table 5.

Table 5: Table of Comparison of the 28th Compressive Strength Values

Expt. No.	Scheffe's (5,2) Polynomial model **(Nwachukwu & others(2017))**		Scheffe's (5,3) Polynomial model **(Present study)**		**Percentage Difference**
	Response Symbol	Average Strength (Nmm^{-2})	Response Symbol	Average Strength (Nmm^{-2})	
1.	Y_1	16.89	Y_1	16.89	0.000
2.	Y_2	14.93	Y_2	16.03	0.011
3.	Y_3	16.80	Y_3	15.26	0.015
4.	Y_4	19.91	Y_4	19.91	0.000
5.	Y_5	19.65	Y_5	19.65	0.000
6.	Y_{12}	16.09	Y_{112}	16.00	0.001
7.	Y_{13}	20.71	Y_{113}	14.37	0.063
8.	Y_{14}	15.20	Y_{122}	15.20	0.000
9.	Y_{15}	16.09	Y_{133}	15.26	0.009
10.	Y_{23}	13.51	Y_{114}	14.00	0.005
11.	Y_{24}	14.84	Y_{144}	14.84	0.000
12.	Y_{25}	16.80	Y_{115}	16.80	0.000
13.	Y_{34}	18.66	Y_{155}	18.06	0.006
14.	Y_{35}	14.40	Y_{223}	14.40	0.000
15.	Y_{45}	16.09	Y_{233}	16.09	0.000
16.			Y_{224}	16.59	0.022
17.			Y_{244}	14.74	0.147
18.			$Y2_{25}$	16.67	0.167
19.			Y_{255}	21.82	0.218

20.	Y_{334}	19.63	0.196
21.	Y_{344}	16.15	0.162
22.	Y_{335}	15.26	0.153
23.	Y_{355}	14.78	0.148
24.	Y_{445}	16.52	0.165
25.	Y_{455}	17.11	0.171
26.	Y_{123}	17.33	0.173
27.	Y_{124}	17.93	0.179
28.	Y_{125}	16.96	0.170
29.	Y_{134}	16.59	0.166
30.	Y_{135}	16.86	0.169
31.	145	17.43	0.174
32.	Y_{234}	17.59	0.176
33.	Y_{235}	17.70	0.177
34.	Y_{245}	18.00	0.180
35.	Y_{345}	18.29	0.183

4.6. DISCUSSION OF RESULTS

Using Scheffe's (5,3) simplex model the values of the compressive strength were obtained. The model gave highest compressive strength of 21.82 Nmm^{-2} corresponding to mix ratio of 0.7:1:1:1.8:1.2 for water, cement, fine and coarse aggregate and glass fibre respectively. The maximum strength using Scheffe's (5,2) simplex model was obtained as 20.71Nmm^{-2} corresponding to mix ratio of 0.59:1:1.45:1.85:0.75. The maximum strength values from both models were greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete. Using the model, compressive strength of all points in the simplex can be derived.

5. CONCLUSION AND RECOMMENDATION

5. 1. CONCLUSION

Scheffe's third degree polynomial (5,3) was used to formulate a model for predicting the compressive strength of GFRC cubes. This model could predict the compressive strength of the GFRC concrete cubes if the mix ratios are known and vice versa. The strengths predicted by the models were in good agreement with the corresponding experimentally observed results. The optimum attainable compressive strength predicted by the Scheffe's (5,3) model at the 28th day was 21.82 N/mm^{2}. When compared with optimum attainable compressive strength predicted by the Scheffe's (5,2) model, given as 20.71 N/mm^2 by Nwachukwu and others (2017), it can be deduced that the strength predicted by Scheffe's (5,3) model is slightly higher than that by Scheffe's (5,2) model. However, the strength predicted by both models meet the minimum standard requirement stipulated by American Concrete Institute of 20N/mm^2 for the compressive strength. With the model, any desired strength of Glass Fibre Reinforced Concrete, given any mix proportions is easily evaluated.

5. 2. RECOMMENDATION

Though the maximum strength actualized is too low compared to other literatures it can also be said that if GFRC is used in the right form, it would produce the optimum strength required for good concrete. The audience are therefore advised to use optimized GFRC for construction purposes, especially light weight structures when economy and safety advantages are considered most. This is due to the fact that replacement of the conventional steel reinforcement with homogenous tiny strands of Alkaline Resistant (AR) glass fibre goes a long way to save cost, as steel reinforcements are more costly than glass fibres.

REFERENCES

ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544.1R-82)", *Concrete International: Design and Construction*. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.

Aggarwal, M.L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.

British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland Cement; London,

Ezeh, J.C. & Ibearugbulam, O.M. (2009):"Application of Scheffe's Model in Optimization of Compressive

Cube Strength of River Stone Aggregate Concrete"; *International Journal of Natural and Appllied Sciences;* Vol. 5, No. 4,Pp 303 – 308.

Ezeh, J.C., Ibearugbulam, O.M. & Anyaogu, L. (2010a):"Optimization of Compressive Strength of Cement- Sawdust Ash Sandcrete Block using Scheffe's Mathematical Model"; *Journal of Applied Engineering Research.* Vol.4, No.4, Pp 487–494.

Ezeh, J.C., Ibearugbulam, O.M.&Anya, U. C (2010b):"Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe's Simplex Method"; *International Journal of Engineering.* Vol.4, No.4, Pp 471 – 478.

.Ibearugbulam, O.M. (2006):"Mathematical Model for Optimization of Compressive Strength of Periwinkle Shell-Granite Aggregate Concrete";*M.Eng.. Thesis,* Federal University of Technology, Owerri, Nigeria.

Nwachukwu, K.C., Okafor, M. ,Thomas, B., Oputa, A.A., Okodugha,D.A. and Osaigbovo, M.E. . (2017): An Improved Model For The Optimization Of The Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC) Using Scheffe's Second Degree Polynomials, Research journal's Journal of Civil Engineering, vol. 3, No. 2

Nwakonobi, T.U and Osadebe, N.N (2008): "Optimization Model for Mix Proportioning of Clay- Ricehusk- Cement Mixture for Animal Buildings"; *Agricultural Engineering International:the CIGR Ejournal,Manuscript BC 08 007,* Vol x

Neville, A.M. (1981): Properties of Concrete; 3rd edition, Pitman, England.

Obam, S.O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, Nigerian Journal of Technology , Vol. 2, No.25, Pp 1 – 10.

Obam, S.O.(2009):"A Mathematical model for Optimization of Strength of concrete : A case study for shear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International l;* Vol. 5, No.9,Pp 76 – 84.

Onwuka,D.O, Okere, C.E.,Arimanwa, J.I. and Onwuka, S.U.(2011):"Prediction of Concrete Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No.1 Pp.95- 107.

Okere, C.E., (2006):"Mathematical Models for Optimization of the Modulus of rupture of concrete"; *M.Eng. Thesis*, Civil Engineering Department, Federal University of Technology, Owerri.

Rao,M.V.K., Kumar, P.R, and Srinivas, R: (2011):"Effect of Size and Shape of Specimen on Compressive

Strength of Glass Fibre Reinforced Concrete (GFRC)"; *FACTA UNIVERSTATIS series: Architecture and Civil Engineering,* vol.9, No. 1, Pp 1-9

Scheffe, H. (1958): Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol.20, Pp. 344-360.