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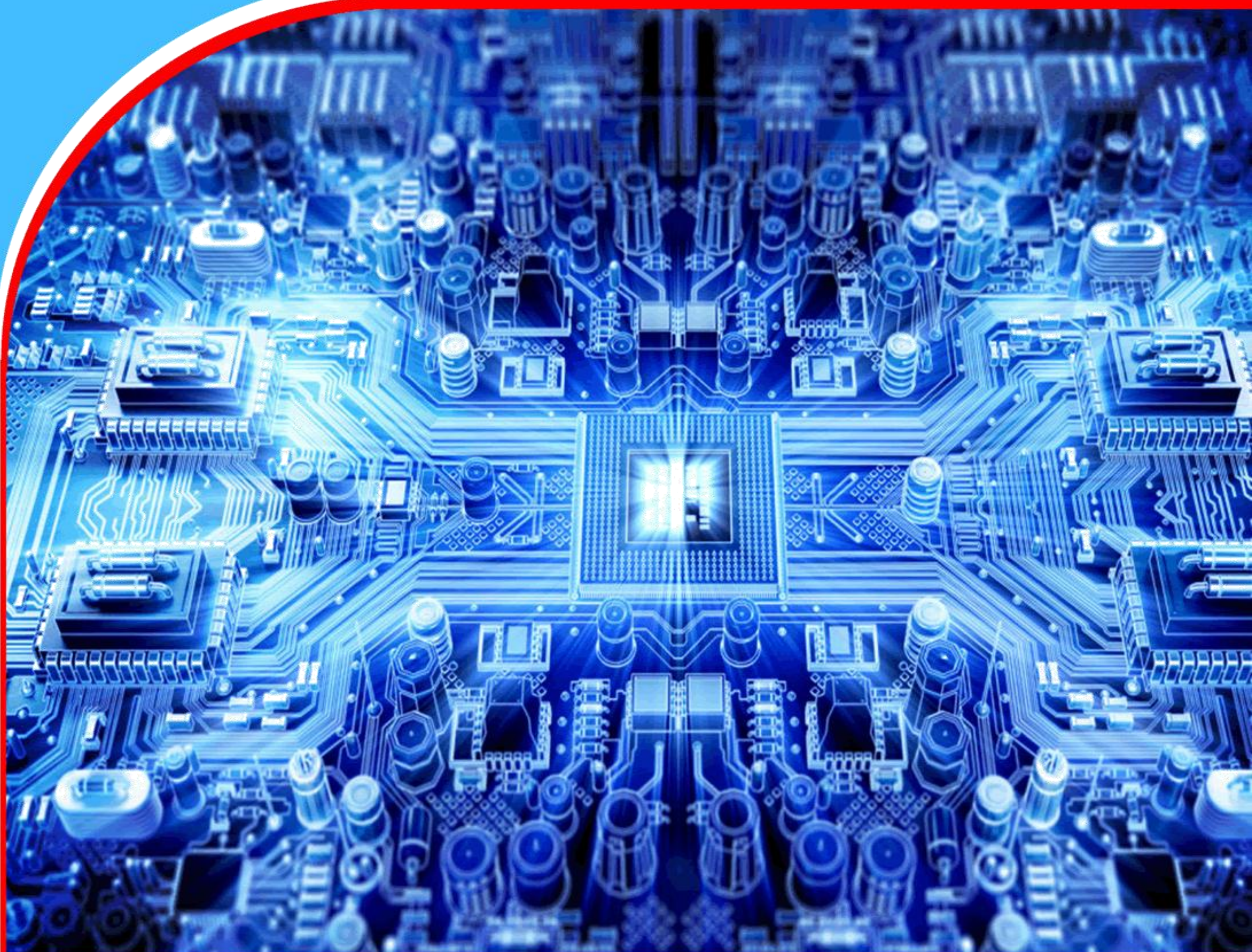

Formulation Of The Total Potential Energy Functional Relevant To The Stability Analysis Of A Doubly Symmetric Single (DSS) Cell Thin- Walled Box Column In Line With Raleigh- Ritz Method

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Formulation Of The Total Potential Energy Functional Relevant To The Stability Analysis Of A Doubly Symmetric Single (DSS) Cell Thin- Walled Box Column In Line With Raleigh- Ritz Method

K. C. Nwachukwu[1], J.C.Ezeh[2], H.O.Ozioko[3], J.A.Eiroboyi [4], and D.C.Nwachukwu[5]

[1,2]Department of Civil Engineering, Federal University Of Technology, Owerri, Imo State, Nigeria

[3]Department of Civil Engineering, Michael Okpara University Of Agriculture, Umudike, Abia State, Nigeria

[4]Sterling Oil Exploration & Energy Production Company, Victoria Island, Lagos, Nigeria

[5]Loraj Consult & Innovation Concept, Umuonyeali Mbieri, Imo State, Nigeria

Corresponding Author's Email: knwachukwu@gmail.com

Abstract

Purpose: This work is concerned with the formulation of peculiar Total Potential Energy Functional (TPEF) for a Doubly Symmetric Single (DSS) cell Thin -walled Box Column (TWBC). The formulated Energy Functional Equations support the stability analysis of a DSS cell thin-walled box (closed) column cross-section using Raleigh - Ritz Method (RRM) with polynomial shape functions.

Methodology: This present formulation is based on the governing TPEF developed by Nwachukwu and others (2017). The polynomial shape functions (only the first two coordinate polynomial shape functions) for different boundary conditions were generated first, and then followed by the formulation of TPEF for different boundary conditions of the DSS cell TWBC.

Findings: The Raleigh- Ritz based formulated TPEF equations are found suitable, handy and simple to be used in the Flexural(F) , Flexural- Torsional(FT) and Flexural- Torsional-Distortional(FTD) buckling/stability analysis of DSS cell TWBC cross-section where data obtained (critical bulking loads) will be compared with the works of other authors in subsequent papers.

Conclusion: Henceforth it is recommended that additional work should be done using more than first two coordinate polynomial shape functions in order to increase the accuracy of RRM.

Keywords: *Doubly Symmetric Single (DSS) Cell ,Total Potential Energy Functional(TPEF), Thin -Walled Box Column(TWBC), Raleigh- Ritz Method(RRM),Bulkling/ Stability Analysis*

1. INTRODUCTION

According to Simao and Simoes da silva (2004a), the use of very slender thin-walled cross-sections members have become increasingly common due to their high stiffness/weight ratio, in recent years. For about a century many branches of the industry have sought stronger and at the same time lighter structural solutions which optimize the effectiveness and the cost of the structures (Andreassen, 2012). Among these are the civil, mechanical, naval, and aerospace industries. This has led to an increasing use of thin-walled structures such as cold-formed steel beams and columns, steel and concrete box girders, ship hulls, trapezoidal steel sheeting and other structures in which one dimension is small compared to the other dimensions. Thin-walled structures such as beams, columns, plates, shells, sheeting, and pipes, among others are frequently used in civil, naval, space, offshore and aerospace constructions.

From the stand point of torsion resistance, Thin-Walled Sections (TWS) in general, are classified into three types:

(a) Open Thin-Wall (OTW); in which a cell shear flow circuit cannot be established in the cross section. Examples are shown in figure 1.

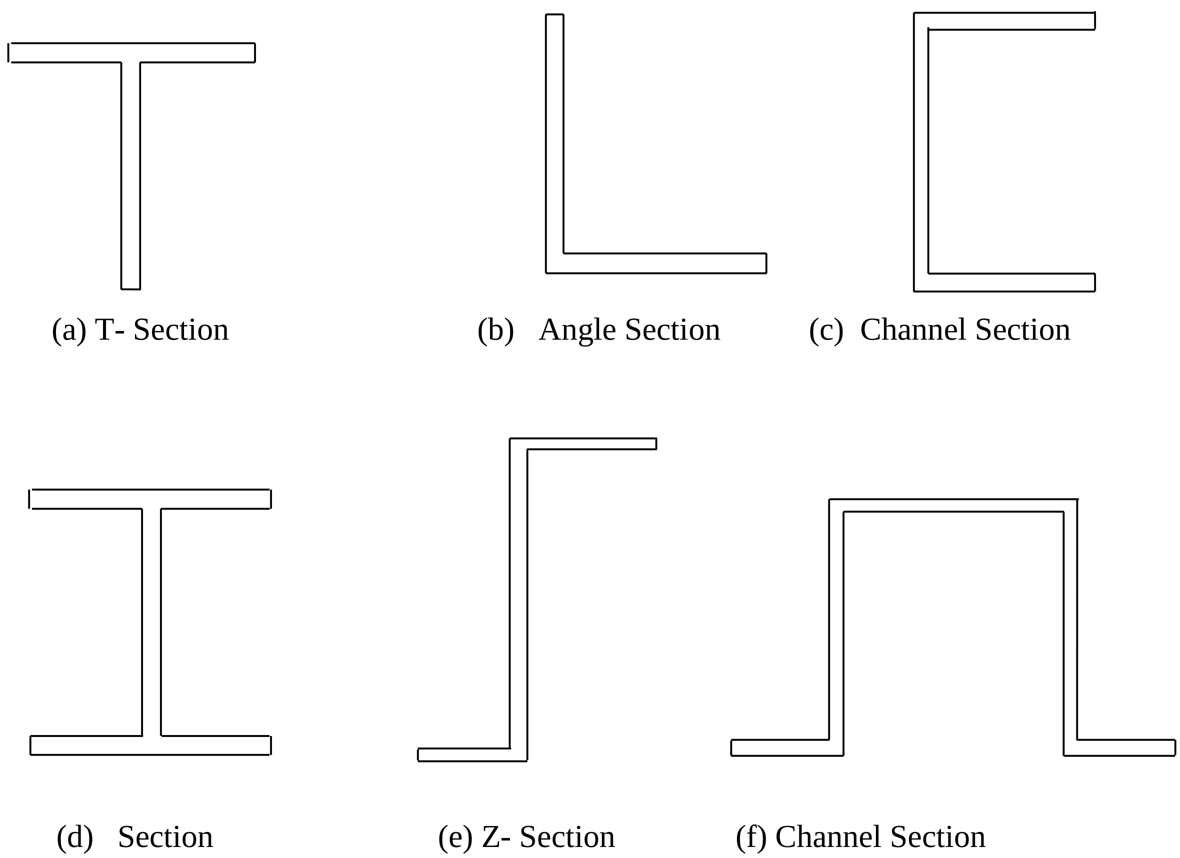
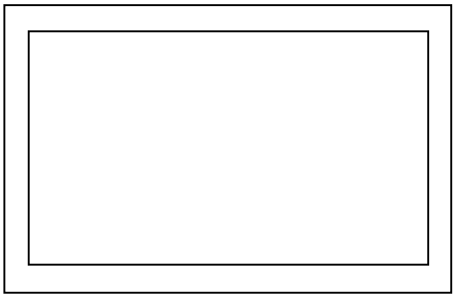
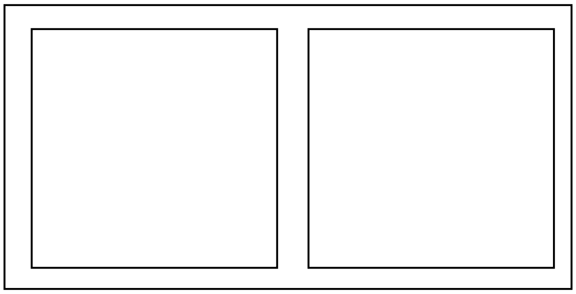


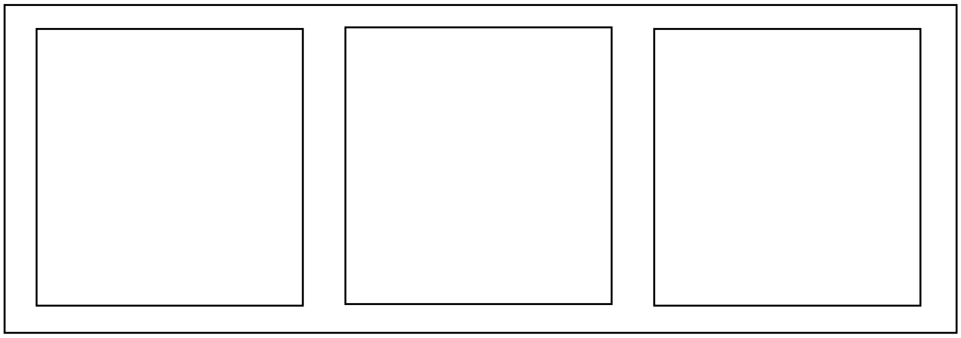
Figure 1: Examples of Thin-Walled Open Cross-Sections

(b) Closed Thin- walled (CTW); in which at least one cell shear flow circuit can be established in the cross- section. Closed TW sections are in turn classified into single cell or multi-cell, according to whether one or several shear flow circuits, respectively can be identified. Examples of closed Thin-Walled sections are shown in figure 2.





(a) Single cell closed cross section (b) double cell closed cross-section



(c) Triple cell closed cross-section

Figure 2: Examples of Thin- walled closed cross sections

(c) Hybrid Thin- Walled (HTW): This type of section contains a mixture of Closed Thin-walled (CTW) and Open Thin-Walled (OTW) components. An example of HTW cross section is shown in figure 3.

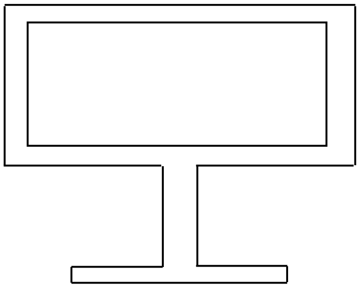
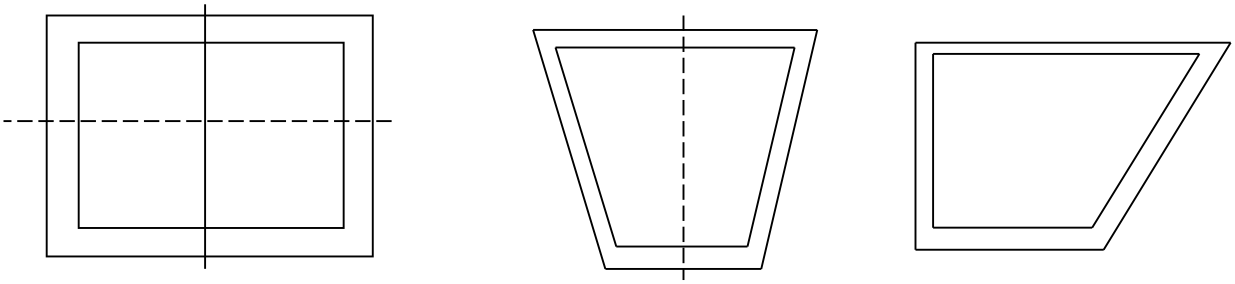


Figure 3: An example of Hybrid Thin- walled cross section

Thin-walled closed cross-section, especially the box typed structures can also have different types of arbitrary cross sections. Different types of arbitrary box cross- section are shown in figure 4.



(a)Doubly symmetric box section (b) Mono-symmetric box section (c) Asymmetric box section

Figure 4: Different Types of Arbitrary Box Cross-Section

Nwachukwu et al., (2017) derived the governing equation for the TPEF for a TWBC applicable to RRM. It becomes expedient to develop the individual or peculiar TPEF for the stability analysis of different thin walled box column cross- sections. Thin- Walled Box Column cross sections that can be analyzed are as listed under**.**

- Doubly Symmetric Single Cell Cross- Section (DSS)
- Doubly Symmetric Multi- Cell Cross- Section (DSM)
- Mono- Symmetric Single Cell Cross- Section (MSS)
- Mono- Symmetric Multi- Cell Cross- Section (MSM)
- Asymmetric Single Cell Cross- Section (ASS)
- Asymmetric Single Cell Cross- Section (ASM)

However, in this work, attention is focused only on the development of TPEF for DSS cross-section. Many researchers have carried out one form of analysis or the other on thin- walled box columns and related topics, but none has used RRM approach with polynomial shape function. For instance, Krolak et al., (2009) presented a theoretical, numerical and experimental analysis of the stability and ultimate load of multi-cell thin-walled columns of rectangular and square cross-sections subjected to axial compression. Shanmugam et al., (1989) presented a numerical method to investigate the ultimate strength behavior of thin-walled steel box columns subjected to axial loads and biaxial end moments. The work of Ezeh (2009) involved a theoretical formulation based on Vlasov's theory as modified by Varbanov, in analyzing flexural, flexural-torsional, and flexural-torsional-distortional buckling modes of thin-walled closed columns. Chidolue and Osadebe (2012), also used Vlasov's theory to carryout Torsional- Distortional analysis of thin- walled box girder bridges. Chidolue and Aginam (2012) investigated the effects of shape factor on the Flexural- Torsional-Distortional behavior of thin- walled box girder structures using Vlasov's Theory. Ezeh (2010) also investigated the buckling behavior of axially compressed multi- cell doubly symmetric thin- walled column using Vlasov's theory. The works of Osadebe and Chidolue (2012a), Osadebe and Chidolue (2012b), Osadebe and Ezeh (2009a), Osadebe and Ezeh (2009b) were also based on Vlasov's method.

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Thus in the area of stability analysis of thin-walled box (closed) columns, little or no effort has been done to use the method of Raleigh- Ritz for analysis. Henceforth, it has become important to further the frontier of knowledge by developing the specific/peculiar TPEF for different DSS boundary conditions in line with RRM. The work is purely theoretical and it is based on Raleigh-Ritz Theory. This work is therefore an attempt to formulate a working TPEF for DSS cross-section based on the developed governing TPEF Equations. The formulated energy functional will now be used to analyze a DSS thin- walled box (closed) columns of different boundary conditions in subsequent papers.

2. GENERATION OF POLYNOMIAL SHAPE FUNCTION

Let us recall the general(governing) Total Potential Energy Functional derived by Nwachukwu et al., (2017) as stated in equation (1).

$$\pi = k_1 \int_L v^2\, x^2 (2L - x)^2 dx + k_2 \int_L (v^I)^2\, dx + k_3 \int_L (v^{II})^2\, dx - k_4 \int_L (v^I)^2\, dx. \quad (1)$$

Where, v is the displacement function, which is a function of polynomial shape function, ϕ

$$k_1 = \frac{Ap^2}{8EI^2}; \quad k_2 = \frac{AG}{2}; \quad k_3 = \frac{E\,I}{2}; \; and \quad k_4 = \frac{P}{2} \quad 2(a\text{-}d)$$

P is critical buckling load, A is Cross sectional area, E is young modulus of elasticity, G is shear modulus, I is moment of inertia, and L is length of the column.

In general, the Rayleigh – Ritz method is stated as:

$$\bar{v} = \sum_i^n c_i\, \phi_i = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \ldots\ldots + c_n\phi_n \quad (3)$$

where $\bar{v}$ = v = assumed lateral displacement of the column/ displacement function

ϕ = Arbitrary function satisfying the boundary conditions of the Column, otherwise known as the polynomial shape function

c = undetermined coefficient / unknown constant.

Our interest is this section is to generate the polynomial shape function, ϕ

2.1. BRIEF PROCEDURES FOR GENERATING THE POLYNOMIAL SHAPE FUNCTIONS

The following steps are necessary for the generation of the polynomial shape function, ϕ.

(i). In general, the coordinate polynomial shape function can be defined in terms of Taylor-Mclaurin series given by:

$$\phi_{i(i=\;1,2\ldots)} = \sum_{i=0}^{nb} a_i\, x^i = a_0 + a_1\, x^1 + a_2\, x^2 + \ldots + a_{nb} x^{nb} \quad (4)$$

where a_i = unknown coefficient and nb = number of boundary conditions that apply to mode of deformation.

(ii). The first coordinate polynomial shape function, ϕ_1 is expected to have the highest order, nb while the second polynomial shape function, ϕ_2 is expected be one order higher, and so on.

(iii). As the efficiency of RRM depends strongly on the correct choice of the coordinate function, the choice of orthonormal functions, ϕ_i over the member's length defined by chen and Lui (1987) as:

$$\int_0^L \phi_i \phi_j \, dx = \begin{cases} c \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

(5)

Accelerates the convergence of the method. In equation (5), the first condition is a normalization rule, where c is a real positive constant usually taken equal to 1 or to the member's length, L, whereas the second condition constitutes an orthogonality condition.

(iv). By applying the selected boundary conditions into equation 5, the process of finding the unknown coefficients, a_i is started.

(v). The normality rule of equation (5) wrt the member's length is now imposed in order to evaluate the remaining coefficients. Note that it is only the positive root of the normalization condition that is chosen.

(vi) By solving for the unknown coefficients, $a_{i,}$ the first coordinate polynomial shape function, ϕ_1 is now completely defined.

(vii) The second coordinate polynomial, ϕ_2 is set up in the same way as the first polynomial, but is one order higher.

(viii).Like the previous polynomial, the selected boundary conditions are used to start up the process of evaluating the unknown coefficients.

(ix) The orthogonality condition is imposed between ϕ_1 (which is already known) and ϕ_2 in the form of

$$\int_0^L \phi_1 \phi_2 \, dx = 0$$

(6)

(x). Next, the normalization condition is now applied in order to evaluate the remaining coefficients of $\phi_{2,}$ using

$$\int_0^L \phi_2^2 \, dx = L$$

(7) After which, the polynomial ϕ_2 is completely defined.

(xi). For this present work, only the first two coordinate polynomial shape functions will be generated and used subsequently.

2.2. GENERATION OF POLYNOMIAL SHAPE FUNCTIONS FOR DIFFERENT BOUNDARY CONDITIONS

The different boundary conditions that will be considered are:

i. Pinned-Pinned (S-S) boundary condition
ii. Fixed-Fixed or Clamped- Clamped(C-C) boundary condition
iii. Fixed – Pinned (C-S) boundary condition

2.2.1. THE POLYNOMIAL SHAPE FUNCTIONS FOR THE PINNED- PINNED (S-S) BOUNDARY CONDITION

For thin- walled columns, simply supported at both ends (S-S), ie the end sections are free to warp, the boundary conditions are:

i., Kinematic conditions:

$\phi_{(x=0)} = 0$ and $\phi_{(x=L)} = 0$
8(a-b)

ii. Static conditions:

$\phi^{II}_{(x=0)} = 0$ and $\phi^{II}_{(x=L)} = 0$
9(a-b)

Using only the kinematic boundary conditions, the first coordinate polynomial shape function is of second order and has three unknown coefficients (see Eqn.4) as follows:

$$\phi_1 = a_o + a_1x + a_2x^2$$
(10)

For $\phi_{1(x=0)} = 0$, we have

$$a_o = 0$$
(11)

for $\phi_{1(x=L)} = 0$, we have

$$\phi_1 = 0 = a_o + a_1L + a_2L^2$$

$$\Rightarrow \quad a_1L + a_2L^2 = 0$$
(12)

solving, $a_1 = - a_2L$
(13)

Imposing the normality rule of Eqn. (5) w.r.t the member's length yields:

$$\int_0^L \phi_1^2 \, dx = L$$
(14)

Where $\phi_1 = - a_2xL + a_2x^2$

$$\therefore \phi_1^2 = a_2^2 x^2L^2 - a_2^2x^3L - a_2^2x^3L + a_2^2x^4$$

$= a_2^2\, x^2L^2 - 2a_2^2x^3L + a_2^2x^4$

(15)

Substituting Eqn. (15) into Eqn. (14) yields:

$$\int_L (a_2^2\, x^2L^2 - 2a_2^2x^3L + a_2^2x^4)dx = L$$

$$\Rightarrow \left[\frac{a_2^2\, L^2x^3}{3} - \frac{2a_2^2Lx^4}{4} + \frac{a_2^2x^5}{5}\right]_0^L = L$$

$$\Rightarrow \frac{a_2^2\, L^5}{3} - \frac{2a_2^2L^5}{4} + \frac{a_2^2L^5}{5} = L$$

Thus $a_2^2 = \frac{30}{L^4}$

$$\Rightarrow \quad a_2 = \pm \frac{\sqrt{30}}{L^2}$$

Taking only the positive roots,

$$a_2 = \frac{\sqrt{30}}{L^2}$$

(16)

Thus $a_1 = -a_2L = -\frac{\sqrt{30}}{L^2}$

(17)

Thus $\phi_1 = a_1x + a_2x^2$

$$= -\frac{\sqrt{30}}{L^2}x + \frac{\sqrt{30}}{L^2}x^2 = \sqrt{30}\left[-\frac{x}{L} + \frac{x^2}{L^2}\right]$$

(18)

The second polynomial which is one order higher than the first polynomial is set in the following form:

$$\Phi_2 = a_o + a_1x + a_2x^2 + a_3x^3$$

(19)

For $\phi_{2(x=0)} = 0$, $a_o = 0$

(20)

For $\phi_{2(x=L)} = 0$, we have

$$\Phi_2 = a_1L + a_2L^2 + a_3L^3 = 0$$

Thus, $a_1 = -a_2L - a_3L^2$

(21)

Substituting Eqn.(21) into Eqn.(19), we have

$$\Phi_2 = -a_2Lx - a_3L^2x + a_2x^2 + a_3x^3$$

(22)

Imposing the orthogonality Rule between Φ_1 of Eqn (18) and Φ_2 of Eqn.(22) gives :

$$\int_0^L \phi_1 \phi_2 \, dx = 0$$

(23)

$$\Rightarrow \int_L \left[a_2 x^2 \sqrt{30} + a_3 L x^2 \sqrt{30} - \frac{a_2 x^3}{L}\sqrt{30} - \frac{a_3 x^4 \sqrt{30}}{L} - \frac{a_2 x^3}{L}\sqrt{30} - a_3 x^3 \sqrt{30} + \frac{a_2 x^4}{L^2}\sqrt{30} + \frac{a_3 x^5}{L^2}\sqrt{30} \right] dx = 0$$

(24)

$$\Rightarrow \left[\frac{a_2 x^3 \sqrt{30}}{3} + \frac{a_3 L x^3 \sqrt{30}}{3} - \frac{a_2 x^4}{4L}\sqrt{30} - \frac{a_3 x^5 \sqrt{30}}{5L} - \frac{a_2 x^4}{4L}\sqrt{30} - \frac{a_3 x^4 \sqrt{30}}{4} + \frac{a_2 x^5}{5L^2}\sqrt{30} + \frac{a_3 x^6}{6L^2}\sqrt{30} \right]_0^L = 0$$

(25)

$$\Rightarrow \frac{a_2 L^3 \sqrt{30}}{3} + \frac{a_3 L^4 \sqrt{30}}{3} - \frac{a_2 L^3}{4}\sqrt{30} - \frac{a_3 L^4 \sqrt{30}}{5} - \frac{a_2 L^3}{4}\sqrt{30} - \frac{a_3 L^4 \sqrt{30}}{4} + \frac{a_2 L^3}{5}\sqrt{30} + \frac{a_3 L^4}{6}\sqrt{30} = 0$$

(26) Simplifying further gives:

$$\frac{a_2 L^3 \sqrt{30}}{3} - \frac{a_2 L^3}{4}\sqrt{30} - \frac{a_2 L^3}{4}\sqrt{30} + \frac{a_2 L^3}{5}\sqrt{30} + \frac{a_3 L^4 \sqrt{30}}{3} - \frac{a_3 L^4 \sqrt{30}}{4} - \frac{a_3 L^4 \sqrt{30}}{5} + \frac{a_3 L^4}{6}\sqrt{30} = 0$$

(27) $\Rightarrow \quad 2a_2 L^3 \sqrt{30} + 3a_3 L^4 \sqrt{30} = 0$

$\therefore \quad 2a_2 L^3 = -3a_3 L^4$

$\therefore \quad a_2 = \frac{-3a_3 L}{2}$

(28) Now substituting Eqn. (28) into Eqn. (22)

$\Phi_2 = -a_2 L x - a_3 L^2 x + a_2 x^2 + a_3 x^3$

$= \frac{3a_3 L^2 x}{2} - a_3 L^2 x - \frac{3a_3 L x^2}{2} + a_3 x^3$

$\Phi_2 = \frac{a_3 L^2 x}{2} - \frac{3a_3 L x^2}{2} + a_3 x^3$

(29) To obtain ϕ_2^2 , we square Eqn.(29)

$$\phi_2^2 = \frac{a_3^2 L^4 x^2}{4} - \frac{3a_3^2 L^3 x^3}{4} + \frac{a_3 L^2 x^4}{2} - \frac{3a_3^2 L^3 x^3}{4} + \frac{9a_3^2 L^2 x^4}{4} - \frac{3a_3^2 L x^5}{2} + \frac{a_3^2 L^2 x^4}{2} - \frac{3a_3^2 L x^5}{2} + a_3^2 x^6$$

(30) Imposing the normality rule of Eqn.(14) gives:

$$\int_0^L \phi_1^2 \, dx = L$$

(31)

$$\Rightarrow \frac{a_3^2 L^7}{12} - \frac{3a_3^2 L^7}{16} + \frac{a_3 L^7}{10} - \frac{3a_3^2 L^7}{16} + 9a_3^2 L^7 - \frac{3a_3^2 L^7}{12} + \frac{a_3^2 L^7}{10} - \frac{3a_3^2 L^7}{12}$$

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$$+ \frac{a_3^2 x^7}{7} = L$$

(32)

$$\Rightarrow \frac{a_3^2 L^7}{840} = L$$

$$\therefore a_3^2 = \frac{840}{L^6}$$

$$\therefore a_3 = \frac{\sqrt{840}}{\sqrt{L^6}} = \frac{\sqrt{210} * \sqrt{4}}{L^3}$$

$$\therefore a_3 = \frac{2 * \sqrt{210}}{L^3}$$

(33) Substituting Eqn. (33) into Eqn. (29), the second coordinate polynomial shape function can now be fully evaluated:

That is; $\Phi_2 = \frac{2* \sqrt{210}\, L^2 x}{2L^3} - \frac{3*2*\sqrt{210}\, Lx^2}{2L^3} + \frac{2*\sqrt{210}\, x^3}{L^3}$

$$= \sqrt{210}\frac{x}{L} - 3\sqrt{210}\frac{x^2}{L^2} + 2 * \sqrt{210}\frac{x^3}{L^3}$$

$$\Phi_2 = \sqrt{210}\left[\frac{x}{L} - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right]$$

(34)

2.2.2. THE POLYNOMIAL SHAPE FUNCTIONS FOR THE FIXED- FIXED (C-C) BOUNDARY CONDITIONS.

For columns, clamped at both ends (C-C columns), without the allowance of warping at the edge sections, the relevant kinematic boundary conditions are:

i. $\phi_{(x=0)} = 0; \ \phi_{(x=L)} = 0$

(35 a-b)

ii. $\phi'_{(x=0)} = 0; \ \phi'_{(x=L)} = 0$

(36 a-b)

Using the four kinematic boundary conditions, the first coordinate polynomial shape function is of 4th order and has five unknown coefficients.

Expanding Eqn. (4) to the 4th order yields:

$$\phi_1 = a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

(37)

Similarly, the second polynomial shape function is of 5th order and has six unknown coefficients. It is set in the following form from Eqn. (4) :

$$\Phi_2 = a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

(38) Evaluating Eqn. (37) and Eqn. (38) following the procedures used for the pinned - pinned boundary conditions, we obtain :

$$\phi_1 = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

$$= \frac{3*\sqrt{70}\,x^2}{L^2} - \frac{6*\sqrt{70}\,x^3}{L^3} + \frac{3*\sqrt{70}x^4}{L^4}$$

(39)

$$= \sqrt{70}\ [\ \frac{3*\ x^2}{L^2} - \frac{6*\ x^3}{L^3} + \frac{3*x^4}{L^4}\ ]$$

(40)

And

$$\Phi_2 = \sqrt{770}\ [\ \frac{-3*\ x^2}{L^2} + \frac{12*\ x^3}{L^3} - \frac{15*x^4}{L^4} + \frac{6*x^5}{L^5}\ ]$$

(41)

2.2.3. THE POLYNOMIAL SHAPE FUNCTIONS FOR THE FIXED- PINNED (C-S) BOUNDARY CONDITIONS.

For columns clamped at one end and simply supported at the other end, the following boundary conditions are fundamental**.**

i. Kinematic conditions:

$$\phi_{(x=0)} = 0\ ;\ \phi_{(x=L)} = 0 \text{ and } \phi'_{(x=0)} = 0;$$ (42a-c)

ii. Static conditions:

$$\phi''_{(x=L)} = 0$$

(43)

By using both the kinematic and static conditions, the first coordinate polynomial shape function is of 4th order and has five unknown coefficients. Expanding Eqn.(4) to the 4th order yields:

$$\Phi_1 = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

(44)

Solving, we obtain that

$$\Phi_1 = \sqrt{\frac{70}{19}}\ [\ \frac{9*\ x^2}{L^2} - \frac{15*\ x^3}{L^3} + \frac{6*x^4}{L^4}\ ]$$

(45)

Similarly, the second coordinate polynomial shape function is found out to be

$$\Phi_2 = \sqrt{\frac{770}{247}}\ [\ \frac{-3*\ x^2}{L^2} + \frac{141*\ x^3}{L^3} - \frac{159*x^4}{L^4} + \frac{21*x^5}{L^5}\ ]$$

(46)

3. FOMULATION OF TOTAL POTENTIAL ENERGY FUNCTIONAL (TPEF) FOR DSS TWBC

CROSS-SECTION

3.1. CROSS SECTIONAL PROPERTIES

Let us recall the general total potential energy functional as stated in Eqn. (1)

$$\pi = k_1 \int_L v^2 x^2 (2L-x)^2 dx + \mathrm{k}_2 \int_L (v^I)^2 dx + k_3 \int_L (v^{II})^2 \mathrm{dx} - k_4 \int_L (v^I)^2 \mathrm{dx}. \quad (47)$$

Where k_1, $k_{2\,,}k_3$ and k_4 are all defined in Eqns.2 (a-d) respectively. In order to compute the cross-section properties in k_1, $k_{2\,,}k_3$ and k_4, we consider the cross- section shown in Fig.5. It is noteworthy that this research work has adopted the same cross -section used by Ezeh (2009) where he employed the use of Vlasov method, in order to allow for easy comparisons.

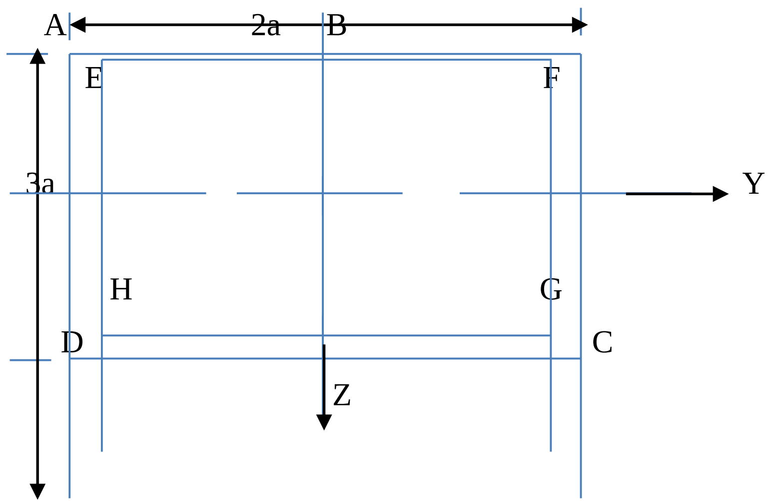

Figure 5: Doubly Symmetric Single Cell Thin – Walled Box Column Cross Section (DSS)

Here, our interest in evaluating the cross sectional properties are to determine the Cross-Sectional Area for DSS, A^{DSS} and its Moment of Inertia, I^{DSS}

Using thin- walled assumptions, points A and E are assumed to be located at A = E, D and H are located at H = D etc. This is illustrated in figure 6.

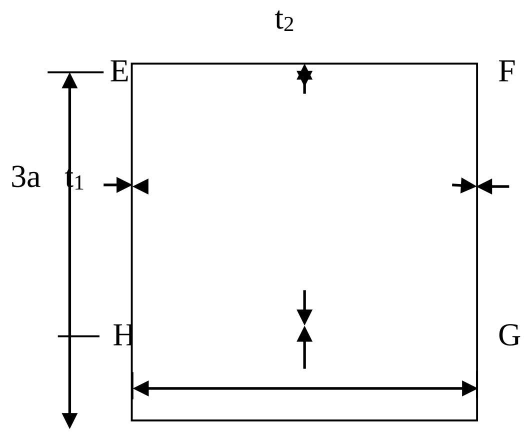

Figure 6: Thin – Walled assumption for DSS

Thus, we obtain our cross- sectional area as

$A^{DSS} = \sum A_i = 3at + 2at + 3at + 2at = 10at$ (48)

As $t_1 = t_2 = t_3 = t_4$ (49)

And our working moment as

$I^{DSS} = I_{ZZ} = \frac{10at^3}{12} + 28\, a^3\, t$ (50)

3.2. TPEF FOR DSS DIFFERENT BOUNDARY CONDITION CASES

3.2.1. CASE 1: PINNED –PINNED [S–S] THIN– WALLED BOX COLUMNS

From Eqn. (47), v, the transverse displacement is defined in Eqn.(3). Using only the first two terms,

$v = c_1\phi_1 + c_2\phi_2$ (51)

where ϕ_1 and ϕ_2 are the polynomial shape functions defined in Eqns.(18) and (34) respectively for Pinned –Pinned boundary conditions as shown in figure 7

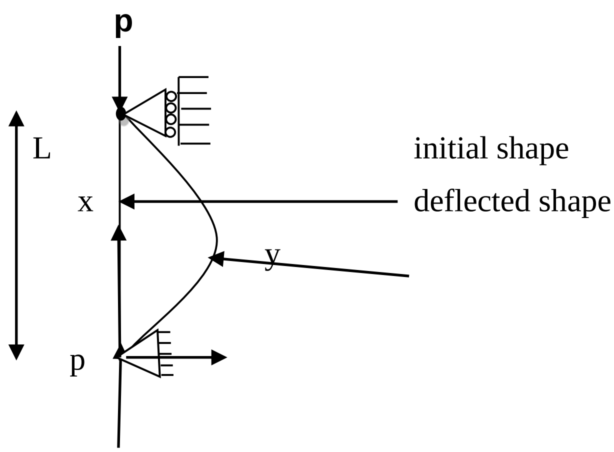

Figure 7: Pinned –Pinned Boundary Condition For Thin– Walled Box Columns

Thus,

$$v = c_1\sqrt{30}\left[-\frac{x}{L} + \frac{x^2}{L^2}\right] + c_2\sqrt{210}\left[\frac{x}{L} - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right]$$ (52)

Let $\beta = \frac{x}{L}$ (53)

$\Rightarrow$ $x = BL$ (54)

$\therefore$ $dx = L\,d\beta$ (55)

Thus Eqn. (52) becomes

$$v = c_1\sqrt{30}[-\beta + \beta^2] + c_2\sqrt{210}\,[\beta - 3\beta^2 + 2\beta^3]$$ (56)

To obtain v^2, we square Eqn.(56)

Thus,

$$v^2 = [-c_1\beta\sqrt{30} + c_1\beta^2\sqrt{30} + c_2\beta\sqrt{210} - 3c_2\beta^2\sqrt{210} + 2c_2\beta^2\sqrt{210}\,]^2$$

$$= 30\,c_1{}^2\beta^2 - 30c_1{}^2\beta^3 - c_1c_2\beta^2\sqrt{6300} + 3c_1c_2\beta^3\sqrt{6300} - 2c_1c_2\beta^4\sqrt{6300}$$

$$- 30\,c_1{}^2\beta^3 + 30\,c_1{}^2\beta^4 + c_1c_2\beta^3\sqrt{6300} - 3c_1c_2\beta^4\sqrt{6300}$$

$$+ 2c_1c_2\beta^5\sqrt{6300} - c_1c_2\beta^2\sqrt{6300} + c_1c_2\beta^3\sqrt{6300} + 210c_2{}^2\beta^3$$

$$+ 30\,c_2{}^2\beta^4(210) - 6\,c_2{}^2\beta^5(210) - 2c_1c_2\beta^4\sqrt{6300} + 2c_1c_2\beta^4\sqrt{6300}$$

$$+2c_1c_2\beta^5\sqrt{6300} + 2\,c_2{}^2\beta^4(210) - 6\,c_2{}^2\beta^5(210) + 4\,c_2{}^2\beta^6(210)$$

(57)

$$v^2 = 30 c_1{}^2\beta^2 - 60c_1{}^2\beta^3 + 30c_1{}^2\beta^4 - 2c_1c_2\beta^2\sqrt{6300} + 8c_1c_2\beta^3\sqrt{6300} -10 c_1c_2\beta^4\sqrt{6300} + 4c_1c_2\beta^5\sqrt{6300} + 210c_2{}^2\beta^2 - 1260c_2{}^2\beta^3 + 2730c_2{}^2\beta^4 - 2520c_2{}^2\beta^5 + 840 c_2{}^2\beta^6$$

(58)

To obtain v', we find the first derivative of Eqn. (56)

$$v' = -c_1\sqrt{30}[-1 + 2\beta] + c_2\sqrt{210}\,[1 - 6\beta + 6\beta^2] = -c_1\sqrt{30} + 2c_1\beta\sqrt{30} + c_2\sqrt{210} - 6c_2\beta\sqrt{210} + 6c_2\beta^2\sqrt{210}$$

(59)

To obtain $(v')^2$, we square Eqn.(59)

$$(v')^2 = [-c_1\sqrt{30} + 2c_1\beta\sqrt{30} + c_2\sqrt{210} - 6c_2\beta\sqrt{210} + 6c_2\beta^2\sqrt{210}]^2$$
$$= c_1{}^2(30) - 2c_1{}^2\beta\,(30) - c_1c_2\sqrt{6300} + 6c_1c_2\beta\sqrt{6300} -6 c_1c_2\beta^2\sqrt{6300} - 2c_1{}^2\beta\,(30) + 4c_1{}^2\beta^2(30) + 2c_1c_2\beta\sqrt{6300} - 12c_1c_2\beta^2\sqrt{6300} + 12c_1c_2\beta^3\sqrt{6300} - c_1c_2\sqrt{6300} + 2c_1c_2\beta\sqrt{6300} + c_2{}^2(210) - 6c_2{}^2\beta\,(210) + 6c_2{}^2(210) + 6c_1c_2\beta\sqrt{6300} -12 c_1c_2\beta^2\sqrt{6300} - 6c_2{}^2\beta\,(210) + 36c_2{}^2\beta^2(210) - 36c_2{}^2\beta^3(210) - 6c_1c_2\beta^2\sqrt{6300} + 12c_1c_2\beta^3\sqrt{6300} + 6c_2{}^2\beta^2(210) - 36c_2{}^2\beta^3(210) + 36c_2{}^2\beta^4(210)$$

(60)

After simplification, $(v')^2$ becomes

$$(v')^2 = 30c_1{}^2 - 120c_1{}^2\beta + 120c_1{}^2\beta^2 - 2c_1c_2\sqrt{6300} + 16c_1c_2\beta\sqrt{6300} -36c_1c_2\beta^2\sqrt{6300} + 24c_1c_2\beta^3\sqrt{6300} + 210c_2{}^2 - 2520c_2{}^2\beta + 10080c_2{}^2\beta^2 - 15120c_2{}^2\beta^3 + 7560c_2{}^2\beta^4$$

(61)

To obtain v^{ll} we evaluate the second derivative of Eqn. (56)

$$v^{ll} = 2c_1\sqrt{30} - 6c_2\sqrt{210} + 12c_2\beta\sqrt{210}$$

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(62)

Squaring Eqn. (62), we have,

$$(v^{ll})^2 = [\, 2c_1 \sqrt{30} - 6c_2 \sqrt{210} + 12c_2\beta \sqrt{210}]^2$$

$$= 4{c_1}^2(30) - 12c_1c_2\sqrt{6300} + 24c_1c_2\beta \sqrt{6300} -$$

$$12c_1c_2\sqrt{6300} + 36{c_2}^2(210) - 72{c_2}^2\beta(210) + 24c_1c_2\beta \sqrt{6300}$$

$$- 72{c_2}^2\beta(210) + 144{c_2}^2\beta^2(210)$$

(63)

$$(v^{ll})^2 = 120{c_1}^2 - 24c_1c_2\sqrt{6300} + 48c_1c_2\beta \sqrt{6300} +$$

$$+ 7560{c_2}^2 - 30240{c_2}^2\beta + 30240{c_2}^2\beta^2$$

(64)

Thus, Eqns. (58), (61) and (64) can now be substituted into Eqn. (47) for further evaluation.

Let us rewrite Eqn. (47) as under:

$$\pi_{DSS}^{S-S} = {k_1}^{DSS}\, {\varphi_1}^{S-S} + {k_2}^{DSS}\, {\varphi_2}^{S-S} + {k_3}^{DSS}{\varphi_3}^{S-S} - {k_4}^{DSS}{\varphi_4}^{S-S}.$$

(65)

Where

$${\varphi_1}^{S-S} = \int_L v^2\, x^2(2L - x)^2 dx$$

(66)

Noting Eqns. (53), (54) and (55), we have

$${\varphi_1}^{S-S} = \int_L v^2\, \beta^2 L^2(2L - \beta L)^2 L d\beta$$

$${\varphi_1}^{S-S} = \int_L\, [4\beta^2 L^5 - 4\beta^3 L^5 + \beta^4\, L^5] v^2 d\beta$$

(67)

Where the symbols, S-S denotes a Pinned- Pinned column, that is simply supported a both ends and DSS denotes Doubly Symmetric Single cell cross-section

$${\varphi_2}^{S-S} = \int_L (v^I)^2\, dx$$

(68)

Also noting Eqns. (53), (54) and (55), we have

$$\mathrm{v}'^{(x)} = \frac{dv}{dx} = \frac{dv^\beta}{Ld\beta} = \frac{1}{L}\mathrm{v}'^{(\beta)}$$

(69)

$$(v^{lx})^2 = \frac{1}{L^2}\, (v^{l\beta})^2$$

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(70)

$$v^{ll(x)} = \frac{1}{L^2}\ v^{ll(\beta)}$$

(71)

And $$(v^{llx})^2 = \frac{1}{L^4}\ (v^{ll\beta})^2$$

(72)

Thus,

$$\varphi_2{}^{S-S} = \int_L \frac{1}{L^2}(v^I)^2\ Ld\beta = \int_L \frac{1}{L}(v^I)^2\ d\beta$$

(73)

$$\varphi_3{}^{S-S} = \int_L (v^{II})^2\ \mathrm{dx}$$

(74)

$$\varphi_3{}^{S-S} = \int_L \frac{1}{L^4}(v^{Il})^2\ Ld\beta = \int_L \frac{1}{L^3}(v^{Il})^2\ d\beta$$

(75)

$$\varphi_4{}^{S-S} = \int_L (v^I)^2\ \mathrm{dx} = \int_L \frac{1}{L}(v^l)^2\ d\beta$$

(76)

Evaluation of $\varphi_1{}^{S-S}$

Substituting Eqn. (58) into Eqn. (67) yields:

$$\varphi_1{}^{S-S} = \int_L\{ [4\beta^2 L^5 - 4\beta^3 L^5 + \beta^4\ L^5][30\ c_1{}^2\beta^2 - 60c_1{}^2\beta^3 + 30c_1{}^2\beta^4 - 2c_1c_2\beta^2\sqrt{6300} + 8c_1c_2\beta^3\sqrt{6300} - 10\ c_1c_2\beta^4\sqrt{6300} + 4c_1c_2\beta^5\sqrt{6300} + 210c_2{}^2\beta^2 - 1260c_2{}^2\beta^3 + 2730c_2{}^2\beta^4 - 2520c_2{}^2\beta^5 + 840\ c_2{}^2\beta^6]\}\ d\beta$$

(77)

$$\varphi_1{}^{S-S} = \int_L\{ 120c_1{}^2\beta^4 L^5 - 240c_1{}^2\beta^5 L^5 + 120c_1{}^2\beta^6\ L^5 - 8\ c_1c_2\beta^4 L^5\sqrt{6300} + 32\ c_1c_2\beta^5 L^5\sqrt{6300} - 40\ c_1c_2\beta^6 L^5\sqrt{6300} + 16\ c_1c_2\beta^7 L^5\sqrt{6300} + 840c_2{}^2\beta^4 L^5$$
$$-5040c_2{}^2\beta^5 L^5 + 10920c_2{}^2\beta^6 L^5 - 10080c_2{}^2\beta^7 L^5 + 3360c_2{}^2\beta^8 L^5 -$$
$$120c_1{}^2\beta^5 L^5 + 240c_1{}^2\beta^6 L^5 - 120c_1{}^2\beta^7\ L^5 + 8\ c_1c_2\beta^5 L^5\sqrt{6300}$$
$$-32\ c_1c_2\beta^6 L^5\sqrt{6300} + 40\ c_1c_2\beta^7 L^5\sqrt{6300} - 16\ c_1c_2\beta^8 L^5\sqrt{6300} - 840c_2{}^2\beta^5 L^5$$
$$+5040c_2{}^2\beta^6 L^5 - 10920c_2{}^2\beta^7 L^5 + 10080c_2{}^2\beta^8 L^5 - 3360c_2{}^2\beta^9 L^5$$
$$+30c_1{}^2\beta^6 L^5 - 60c_1{}^2\beta^7 L^5 + 30c_1{}^2\beta^8\ L^5 - 2c_1c_2\beta^6 L^5\sqrt{6300} + 8\ c_1c_2\beta^7 L^5\sqrt{6300} - 10\ c_1c_2\beta^8 L^5\sqrt{6300}$$
$$+4\ c_1c_2\beta^9 L^5\sqrt{6300} + 210c_2{}^2\beta^6 L^5 - 1260c_2{}^2\beta^7 L^5 + 2730c_2{}^2\beta^8 L^5 - 2520c_2{}^2\beta^9 L^5 +$$

$840{c_2}^2\beta^{10}L^5\}d\beta$

(78)

$$\varphi_1{}^{S-S} = \int_L\{\, 120{c_1}^2\beta^4L^5 - 360{c_1}^2\beta^5L^5 + 390{c_1}^2\beta^6\, L^5 - 180{c_1}^2\beta^7\, L^5 + 30{c_1}^2\beta^8L^5 - 8\, c_1c_2\beta^4L^5\sqrt{6300} + 40\, c_1c_2\beta^5L^5\sqrt{6300} - 744\, c_1c_2\beta^6L^5\sqrt{6300} + 64\, c_1c_2\beta^7L^5\sqrt{6300} - 26\, c_1c_2\beta^8L^5\sqrt{6300} + 4\, c_1c_2\beta^9L^5\sqrt{6300} + 840{c_2}^2\beta^4L^5 - 5880{c_2}^2\beta^5L^5 + 16170{c_2}^2\beta^6L^5 - 22260{c_2}^2\beta^7L^5 + 16170{c_2}^2\beta^8L^5 - 5880{c_2}^2\beta^9L^5 + 840{c_2}^2\beta^{10}L^5]d\beta$$

(79)

Simplifying $\varphi_1{}^{S-S}$ further, we have:

$$\varphi_1{}^{S-S} = \frac{120{c_1}^2L^{10}}{5} - \frac{360{c_1}^2L^{11}}{6} + \frac{390{c_1}^2L^{12}}{7} - \frac{180{c_1}^2L^{13}}{8} + \frac{30{c_1}^2L^{14}}{9} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{40c_1c_2\sqrt{6300}\,L^{11}}{6} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + \frac{64c_1c_2\sqrt{6300}\,L^{13}}{8} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{4c_1c_2\sqrt{6300}\,L^{15}}{10} + \frac{840{c_2}^2L^{10}}{5} - \frac{5880{c_2}^2L^{11}}{6} + \frac{16170{c_2}^2L^{12}}{7} - \frac{22260{c_2}^2L^{13}}{8} + \frac{16170{c_2}^2L^{14}}{9} - \frac{5880{c_2}^2L^{15}}{10} + \frac{840{c_2}^2L^{16}}{11}$$

(80)

Thus ,

$$\varphi_1{}^{S-S} = 24{c_1}^2\, L^{10} - 60{c_1}^2L^{11} + \frac{390{c_1}^2L^{12}}{7} - 10{c_1}^2L^{13} + \frac{10{c_1}^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\,L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + 8\, c_1c_2\sqrt{6300}\,L^{13} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\,L^{15}}{5} + 168{c_2}^2\, L^{10} - 980{c_2}^2L^{11} + 2310{c_2}^2\, L^{12} - \frac{5565{c_2}^2L^{13}}{2} + \frac{5390{c_2}^2L^{14}}{3} - 588{c_2}^2\, L^{15} + \frac{840{c_2}^2L^{16}}{11}$$

(81)

Evaluation of $\varphi_2{}^{S-S}$

Substituting Eqn. (61) into Eqn. (73) yields:

$$\varphi_2{}^{S-S} = \int_L\{\tfrac{1}{L}[30{c_1}^2 - 120{c_1}^2\beta + 120{c_1}^2\beta^2 - 2\, c_1c_2\sqrt{6300} + 16\, c_1c_2\beta\sqrt{6300} - 36\, c_1c_2\beta^2\sqrt{6300} + 24\, c_1c_2\beta^3\sqrt{6300} + 210{c_2}^2 - 2520{c_2}^2 + 10080{c_2}^2\beta^2 - 15120{c_2}^2\beta^3 + 7560{c_2}^2\beta^4\,]\,\}$$

(82)

Integrating Eqn. (82) gives:

$$\varphi_2{}^{S-S} = \frac{1}{L}[30{c_1}^2\beta - \frac{120{c_1}^2\beta^2}{2} + \frac{120{c_1}^2\beta^3}{3} - 2\, c_1c_2\beta\sqrt{6300} + \frac{16\, c_1c_2\beta^2\sqrt{6300}}{2} - \frac{36\, c_1c_2\beta^3\sqrt{6300}}{3} + \frac{24\, c_1c_2\beta^4\sqrt{6300}}{4}$$

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$$+ 210c_2{}^2\beta - \frac{2520c_2{}^2\beta^2}{2} + \frac{10080c_2{}^2\beta^3}{3} - \frac{15120c_2{}^2\beta^4}{4} + \frac{7560c_2{}^2\beta^4}{5} \Big]_0^L$$
(83)

$$\varphi_2{}^{S-S} = \frac{1}{L}[30c_1{}^2L - 60c_1{}^2 L^2 + 40c_1{}^2L^3 - 2\, c_1c_2\sqrt{6300}\, L$$
$$+8\, c_1c_2\, \sqrt{6300}\, L^2 - 12c_1c_2\, \sqrt{6300}\, L^3 + 6\, c_1c_2\, \sqrt{6300}\, L^4 + 210c_2{}^2L - 1260c_2{}^2L^2$$
$$+ 3360c_2{}^2L^3 - 3780c_2{}^2L^4 + 1512c_2{}^2L^5]$$
(84)

$$\varphi_2{}^{S-S} = [30c_1{}^2 - 60c_1{}^2 L + 40c_1{}^2L^2 - 2\, c_1c_2\sqrt{6300}$$
$$+8\, c_1c_2\, \sqrt{6300}\, L - 12c_1c_2\, \sqrt{6300}\, L^2 + 6\, c_1c_2\, \sqrt{6300}\, L^3 + 210c_2{}^2 - 1260c_2{}^2L$$
$$+ 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4]$$
(85)

Evaluation of $\boldsymbol{\varphi_3{}^{S-S}}$

Substituting Eqn. (64) into Eqn. (75) yields:

$$\varphi_3{}^{S-S} = \int_L \{ \frac{1}{L^3}[120c_1{}^2 - 24\, c_1c_2\sqrt{6300} + 48\, c_1c_2\beta\sqrt{6300} + 7560c_2{}^2 - 30240c_2{}^2\beta$$
$$+ 30240c_2{}^2\beta^2]\}d\beta$$
(86)

$$\varphi_3{}^{S-S} = \frac{1}{L^3}[120c_1{}^2\beta - 24\, c_1c_2\sqrt{6300}\,\beta + \frac{48\, c_1c_2\beta^2\sqrt{6300}}{2}$$
$$+ 7560c_2{}^2\beta - \frac{30240c_2{}^2\beta^2}{2} + \frac{30240c_2{}^2\beta^3}{3} \Big]_0^L$$
(87)

$$\varphi_3{}^{S-S} = \frac{1}{L^3}[120c_1{}^2L - 24\, c_1c_2\sqrt{6300}\, \mathrm{L} + 24\, c_1c_2\, \sqrt{6300}\, L^2$$
$$+ 7560c_2{}^2L - 15120c_2{}^2L^2 + 10080c_2{}^2L^3]$$
(88)

$$\varphi_3{}^{S-S} = \frac{120c_1{}^2}{L^2} - \frac{24\, c_1c_2\sqrt{6300}}{L^2} + \frac{24\, c_1c_2\, \sqrt{6300}}{L}$$
$$+ \frac{7560c_2{}^2}{L^2} - \frac{15120c_2{}^2}{L} + 10080c_2{}^2$$

(89)

Evaluation of $\varphi_4{}^{S-S}$

From Eqn.(80), we have that:

$$\varphi_4{}^{S-S} = \varphi_2{}^{S-S} = \int_L \frac{1}{L}(v^l)^2 \, d\beta$$

(90)

Thus, from Eqn. (85)

$$\varphi_4{}^{S-S} = [30c_1{}^2 - 60c_1{}^2 L + 40c_1{}^2L^2 - 2\, c_1c_2\sqrt{6300} + 8\, c_1c_2 \sqrt{6300}\, L - 12c_1c_2 \sqrt{6300}\, L^2 + 6\, c_1c_2 \sqrt{6300}\, L^3 + 210c_2{}^2 - 1260c_2{}^2L + 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4]$$

(91)

Thus, the Total Potential Energy Functional for the Pinned-Pinned DSS thin- walled box column, π_{DSS}^{S-S} is obtained by substituting the equations for $\varphi_1{}^{S-S}$, $\varphi_2{}^{S-S}$, $\varphi_3{}^{S-S}$ *and* $\varphi_4{}^{S-S}$ into Eqn. (65) .

That is substituting the Eqns. (81), (85) (89) and.(91) into Eqn. (65) yields:

$$\pi_{DSS}^{S-S} = k_1{}^{DSS}\varphi_1{}^{S-S} + k_2{}^{DSS}\varphi_2{}^{S-S} + k_3{}^{DSS}\varphi_3{}^{S-S} - k_4{}^{DSS}\varphi_4{}^{S-S}$$

$$= k_1{}^{DSS}\Big[24c_1{}^2L^{10} - 60c_1{}^2L^{11} + \frac{390c_1{}^2L^{12}}{7} - 10c_1{}^2L^{13} + \frac{10c_1{}^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\,L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + 8\,c_1c_2\sqrt{6300}\,L^{13} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\,L^{15}}{5} + 168c_2{}^2L^{10} - 980c_2{}^2L^{11} + 2310c_2{}^2L^{12} - \frac{5565c_2{}^2L^{13}}{2} + \frac{5390c_2{}^2L^{14}}{3} - 588c_2{}^2L^{15} + \frac{840c_2{}^2L^{16}}{11}\Big]$$

$$+ k_2{}^{DSS}[30c_1{}^2 - 60c_1{}^2 L + 40c_1{}^2L^2 - 2\, c_1c_2\sqrt{6300} + 8\, c_1c_2 \sqrt{6300}\, L - 12c_1c_2 \sqrt{6300}\, L^2 + 6\, c_1c_2 \sqrt{6300}\, L^3 + 210c_2{}^2 - 1260c_2{}^2L + 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4]$$

$$+ k_3{}^{DSS}\Big[\frac{120c_1{}^2}{L^2} - \frac{24\,c_1c_2\sqrt{6300}}{L^2} + \frac{24\,c_1c_2\sqrt{6300}}{L} + \frac{7560c_2{}^2}{L^2} - \frac{15120c_2{}^2}{L} + 10080c_2{}^2\Big]$$

$$-k_4{}^{DSS}[30c_1{}^2 - 60c_1{}^2 L + 40c_1{}^2L^2 - 2\, c_1c_2\sqrt{6300} + 8\, c_1c_2 \sqrt{6300}\, L - 12c_1c_2 \sqrt{6300}\, L^2 + 6\, c_1c_2 \sqrt{6300}\, L^3 + 210c_2{}^2 - 1260c_2{}^2L$$

$+ 3360{c_2}^2L^2 \ - 3780{c_2}^2L^3 + 1512{c_2}^2L^4 \,]$

(92)

Where

${k_1}^{DSS} = \frac{A^{DSS}p^2}{8EI^{2(DSS)}}$, ${k_2}^{DSS} = \frac{A^{DSS}G}{2}$, ${k_3}^{DSS} = \frac{E\,I^{DSS}}{2}$ & ${k_4}^{DSS} = \frac{P}{2}$

(93 - 96)

A^{DSS} and I^{DSS} are defined in Eqns.(48) and (50) respectively. P is critical buckling load

3.2.2. CASE 2: FIXED-FIXED [C-C] THIN-WALLED BOX COLUMN.

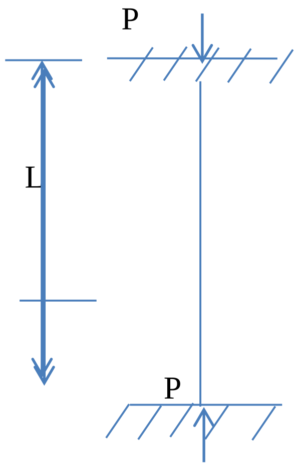

Figure 8: Fixed- Fixed Boundary Condition for Thin- Walled Column.

Here, using Eqn. (51)

$v = c_1\phi_1 + c_2\phi_2$

(97)

where ϕ_1 and ϕ_2 are defined in Eqns.(40) and (41) respectively for fixed –fixed boundary conditions.

Substituting Eqns. (40) and (41) into Eqn.(97) gives:

$v = c_1 [3\sqrt{70}\ \frac{x^2}{L^2} - 6\sqrt{70}\ \frac{x^3}{L^3} + 3\sqrt{70}\ \frac{x^4}{L^4}] + c_2 [- 3\sqrt{770}\ \frac{x^2}{L^2} + 12\sqrt{770}\ \frac{x^3}{L^3} - 15\sqrt{770}\ \frac{x^4}{L^4}$

$+ \ 6\sqrt{70}\ \frac{x^5}{L^5}]$

(98)

Substituting Eqn. (53) into Eqn.(98) gives:

$v = 3c_1\sqrt{70}\ \beta^2 - 6c_1\sqrt{70}\ \beta^3 + 3c_1\sqrt{70}\ \beta^4 - 3c_2\sqrt{770}\ \beta^2$

$+ 12c_2\sqrt{770}\ \beta^3 - \ 15c_2\sqrt{770}\ \beta^4 + 6c_2\sqrt{770}\ \beta^5$

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(99)

And after evaluation and simplification, we obtain that

$$\pi_{DSS}^{C-C} = k_1{}^{DSS}\varphi_1{}^{C-C} + k_2{}^{DSS}\varphi_2{}^{C-C} + k_3{}^{DSS}\varphi_3{}^{C-C} - k_4{}^{DSS}\varphi_4{}^{C-C}$$

$$= k_1{}^{DSS}\,[360c_1{}^2 L^{12} - 1575c_1{}^2L^{13} + 2870c_1{}^2L^{14} - 2772c_1{}^2L^{15} + \frac{16380c_1{}^2L^{16}}{11} - 420c_1{}^2L^{17} + \frac{630c_1{}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960c_2{}^2 L^{12} - 31185c_2{}^2L^{13} + 105490c_2{}^2 L^{14} - 1995840c_2{}^2L^{15} + 230580c_2{}^2 L^{16} - 166320c_2{}^2L^{17} + \frac{949410c_1{}^2L^{18}}{13} - 17820c_2{}^2L^{19} + 1848c_2{}^2L^{20}]$$

$$+k_2{}^{DSS}\,[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 - 600600c_2{}^2L^5 + 633600c_2{}^2L^6 - 346500c_2{}^2L^7 + 77000c_2{}^2L^8]$$

$$+ k_3{}^{DSS}\,[\frac{2520c_1{}^2}{L^2} - \frac{8820c_1{}^2}{L} + 40320c_1{}^2 - 45360c_1{}^2L + 18144c_1{}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L} - 2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,L - 4320c_1c_2\sqrt{53900}L^2 + 1440c_1c_2\sqrt{53900}L^3 + \frac{27720c_2{}^2}{L^2} - \frac{332640c_2{}^2}{L} + 1884960c_2{}^2 - 5266800c_2{}^2L + 7650720c_2{}^2L^2 - 5544000c_2{}^2L^3 + 1584000c_2{}^2L^4]$$

$$-k_4{}^{DSS}\,[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 - 600600c_2{}^2L^5 + 633600c_2{}^2L^6 - 346500c_2{}^2L^7 + 77000c_2{}^2L^8]$$

(100)

Where $k_1{}^{DSS}$, $k_2{}^{DSS}$, $k_3{}^{DSS}$ and $k_4{}^{DSS}$ are defined in Eqns. (93), (94), (95) & (96) respectively.

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3.2.3. CASE 3 : FIXED-PINNED [C-S] THIN-WALLED COLUMN

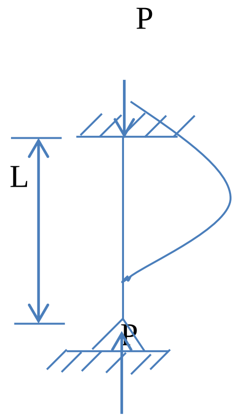

Figure 9: Fixed-Pinned Boundary Condition for Thin- Walled Box Column.

Using Eqn.(51),

$$v = c_1\phi_1 + c_2\phi_2$$
(101)

Where ϕ_1 and ϕ_2 are defined in Eqns.(45) and (46) for Fixed-Pinned Boundary Condition. Substituting Eqns.(45) and (46) into Eqn. (101) yields:

$$v = c_1\left[\frac{9M_1x^2}{L^2} - \frac{15M_1x^3}{L^3} + \frac{6M_1x^4}{L^4}\right] + c_2\left[-\frac{3M_2x^2}{L^2} + \frac{141M_2x^3}{L^3} - \frac{159M_2x^4}{L^4} + \frac{21M_2x^5}{L^5}\right]$$
(102)

$$\text{Where } M_1 = \frac{\sqrt{70}}{\sqrt{19}} \quad \& \quad M_2 = \frac{\sqrt{770}}{\sqrt{247}}$$
(103 - 104)

Substituting Eqn.(53) into Eqn. (102) yields:

$$v = 9c_1M_1\beta^2 - 15c_1M_1\beta^3 + 6c_1M_1\beta^4 - 3c_2M_2\beta^2 + 141c_2M_2\beta^3 - 159c_2M_2\beta^4 + 21c_2M_2\beta^5$$
(105)

And after evaluation and simplifications, we obtain that :

$$\pi_{DSS}^{C-S} = k_1^{DSS}\varphi_1^{C-S} + k_2^{DSS}\varphi_2^{C-S} + k_3^{DSS}\varphi_3^{C-S} - k_4^{DSS}\varphi_4^{C-S}$$

$$= k_1^{DSS}\left[\frac{22680c_1^2L^{12}}{133} - \frac{98280c_1^2L^{13}}{152} + \frac{174510c_1^2L^{14}}{171} - \frac{162540c_1^2L^{15}}{190} + \frac{83790c_1^2L^{16}}{209} - \frac{22680c_1^2L^{17}}{228}\right.$$

$$+ \frac{2520c_1^2L^{18}}{247} - \frac{216}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{12} + \frac{10728}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{13} - \frac{39078}{9}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{14} + \frac{58500}{10}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{15} - \frac{44640}{11}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{16} +$$

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$$\frac{18000}{12}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{17} - \frac{3546}{13}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{18} + \frac{252}{14}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{19}$$

$$+\frac{27720{c_2}^2L^{12}}{1729}$$

$$-\frac{2633400{c_2}^2L^{13}}{1976}+\frac{66784410{c_2}^2L^{14}}{2223}-\frac{203312340{c_2}^2L^{15}}{2470}+\frac{250637310{c_2}^2L^{16}}{2717}-\frac{151295760{c_2}^2L^{17}}{2964}+\frac{45952830{c_2}^2L^{18}}{3211}-\frac{6500340{c_2}^2L^{19}}{3458}+\frac{339570{c_2}^2L^{20}}{3705}]$$

$$+{k_2}^{DSS}[\frac{22680{c_1}^2L^2}{57}-\frac{113400{c_1}^2L^3}{76)}+\frac{202230{c_1}^2L^4}{95}-\frac{151200{c_1}^2L^5}{114}+\frac{40320{c_1}^2L^6}{133}-\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2+\frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3-\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^4+$$

$$\frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^5-\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^6+\frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^7$$

$$+\frac{27720{c_2}^2L^2}{741}-\frac{3908520{c_2}^2L^3}{988}+\frac{143497970{c_2}^2L^4}{1235}-\frac{415273320{c_2}^2L^5}{1482}$$

$$+\frac{379861020{c_2}^2L^6}{1729}-\frac{102841200{c_2}^2L^7}{1976}+\frac{8489250{c_2}^2L^8}{2223}]$$

$$+{k_3}^{DSS}[\frac{22680{c_1}^2}{19L^2}-\frac{226800{c_1}^2}{38L}+\frac{748440{c_1}^2}{57}-\frac{907200{c_1}^2L}{76}+\frac{362880{c_1}^2L^2}{95}-\frac{216}{L^2}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+$$

$$\frac{31536}{2L}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}-\frac{221832}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+\frac{480384}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L-\frac{350352}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2$$

$$+\frac{60480}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3+\frac{24640{c_2}^2}{247L^2}-\frac{7817040{c_2}^2}{494L}+\frac{568731240{c_2}^2}{741}-\frac{2489699520{c_2}^2L}{988}$$

$$+\frac{3350350080{c_2}^2L^2}{1235}-\frac{123094400{c_2}^2L^3}{1482}+\frac{135828000{c_2}^2L^4}{1729}]$$

$$-{k_4}^{DSS}[\frac{22680{c_1}^2L^2}{57}-\frac{113400{c_1}^2L^3}{76)}+\frac{202230{c_1}^2L^4}{95}-\frac{151200{c_1}^2L^5}{114}+\frac{40320{c_1}^2L^6}{133}-\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2$$

$$+\frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3-\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^4+\frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^5-\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^6+$$

$$\frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^7+\frac{27720{c_2}^2L^2}{741}-\frac{3908520{c_2}^2L^3}{988}+\frac{143497970{c_2}^2L^4}{1235}-\frac{415273320{c_2}^2L^5}{1482}$$

$$+\frac{379861020{c_2}^2L^6}{1729}-\frac{102841200{c_2}^2L^7}{1976}+\frac{8489250{c_2}^2L^8}{2223}]$$

(106)

Where ${k_1}^{DSS}$, ${k_2}^{DSS}$, ${k_3}^{DSS}$ and ${k_4}^{DSS}$ are defined in Eqns. (93), (94), (95) & (96) respectively.

4. CONCLUSIONS AND RECOMMENDATON

4.1 CONCLUSIONS

The study was able to formulate peculiar Total Potential Energy Functionals (TPEF) for DSS stability analysis. The formulated Raleigh- Ritz based DSS TPEF given in Eqn.(92) for S-S boundary condition, Eqn.(100) for C-C boundary condition and Eqn.(106) for C-S boundary condition are found handy and convenient to be used in the bulking/stability analysis of DSS cross- sections. The derived expressions will now be used to formulate series of stability matrices in subsequent publications where the critical bulkling load will be evaluated.

4.2 RECOMMENDATION

For this present work, only the first two coordinate polynomial shape functions were generated and used for the formulation of TPEF for DSS stability analysis. Thus it is recommended that additional work should be done using more than first two coordinate polynomial shape functions.

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