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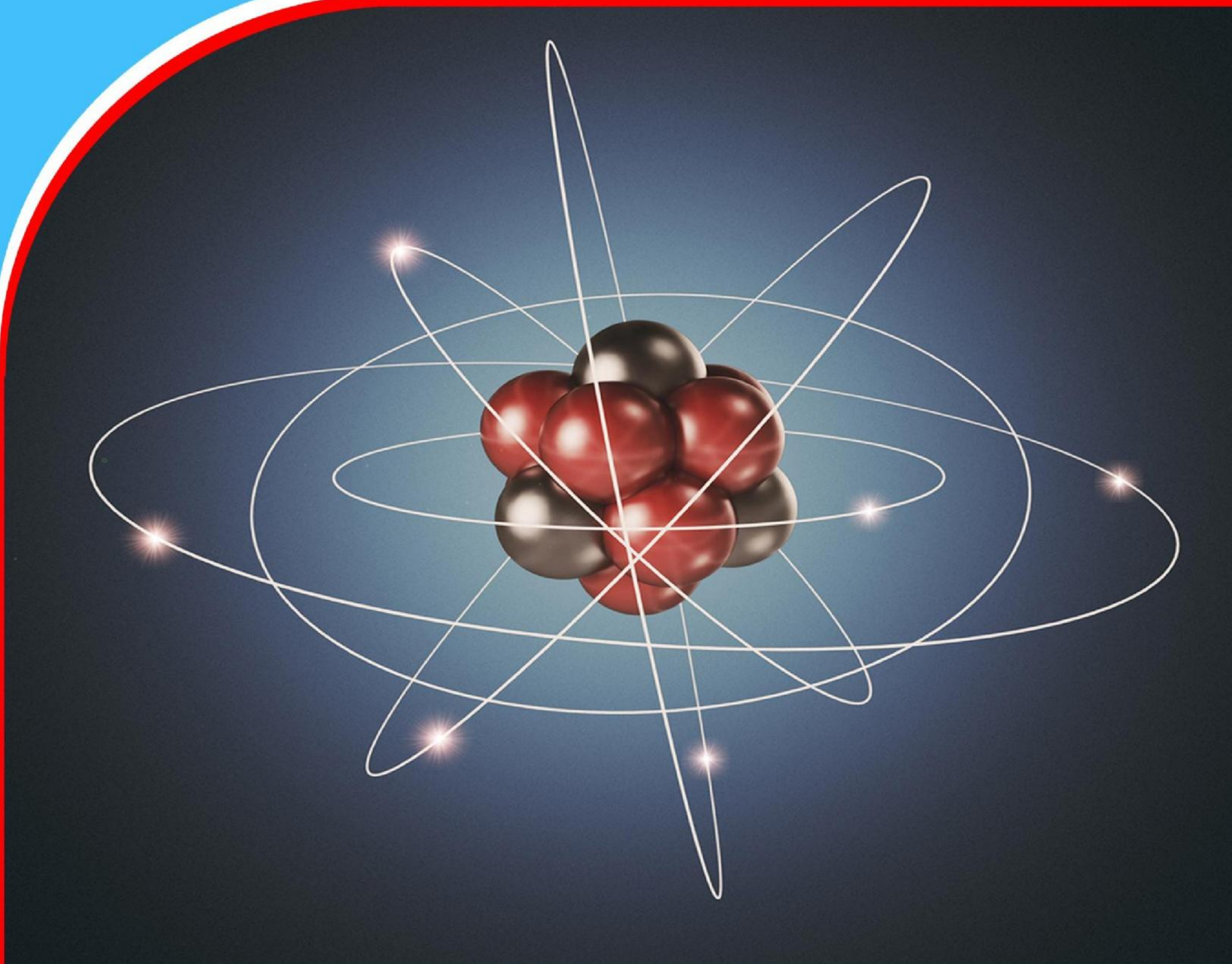

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HYDROMAGNETIC MIXED CONVECTION FLOW OF AN EXOTHERMIC FLUID IN A VERTICAL CHANNEL

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Abstract

In this paper, Hydromagnetic mixed convection flow of an exothermic fluid in a vertical channel is considered. The dimensionless ordinary differential equations were solved using differential transformation method (DTM) to obtain the expression of velocity, temperature and concentration. From momentum, energy and mass equations. The effect of Skin friction, Nusselt number and Sherwood number with various parameters on velocity, temperature and concentration are presented and discussed. The result indicated that the effect of γ_t, is to increase the Skin friction while K increases it at upper plate and suppresses it at lower plate.

Key words: *Differential Transformation Method, Hydromagnetic, Mixed convection, Heat Mass Transfer.*

1. INTRODUCTION

Mixed convection flow in a vertical channel has been the subject of many investigations due to its possible application in many industrial and engineering processes. These include cooling of electronic equipment, heating of the Trombe wall system, gas cooled nuclear reactors and others. Hydromagnetics is the study of the magnetic properties and behavior of electrically conducting fluids. Example of such magnetofluid includes plasmas, liquid metals, salt water and electrolytes. Mixed (combined) convection is a combination of forced and free convections which is the general case of convection when a flow is determined simultaneously by both an outer forcing system (i.e. outer energy supply to the fluid-streamlined body system) and inner volumetric (mass) forces. Ahmad and Jha [1]. Xiao and Tao [12] analyzed the fully laminar fully developed laminar flow and heat transfer in asymmetric wavy channel.

Aung and Worku [3] discussed the mixed convection in duct with asymmetric wall heat and fluxes. Aung and Worku [4] discuss Developing flow and flow reversal in a vertical channel with asymmetric wall temperatures. Sheikholeslami *et al.* [11] discuss the free convection of magnetic nanofluid considering MFD viscosity effect. In most of the chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. These processes take place in numerous industrial applications i.e. polymer production, manufacturing of ceramics, glassware, food processing, etc. Afify [2] studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Jha and Michael [7] discuss the mixed convection flow in a vertical channel with temperature dependent viscosity and flow reversal.

Cheeryal and Keesara [6] studied a numerical solution of mass transfer effects on an unsteady free convection flow of an incompressible electrically conducting viscous dissipative fluid past an infinite vertical porous plate under the influence of a uniform magnetic field considered normal to the plate has been obtained. The effects of flow parameters on the velocity, temperature and concentration fields are presented through the graphs. Chamkha and Kumar [5] established a numerical analysis carried out to study heat mass transfer effects on MHD natural convection flow past an impulsively moving vertical plate with ramped temperature. The dimensionless governing momentum, energy and concentration equations for this investigation are solved numerically by using most flexible method which is finite element method.

The effects of various parameters on the Skin friction, Nusselt number and the Sherwood number are presented. Finally obtained numerical results are compared with the previously published results and are in quantitative agreement. Seth, Sharma and Kumbhakar [9] presented the investigation of Heat and Mass Transfer Effects on Unsteady MHD Natural Convection Flow of a Chemically Reactive and Radiating Fluid through a Porous Medium Past a Moving Vertical Plate with Arbitrary Ramped Temperature is carried out. Exact solutions of momentum, energy and concentration equations are obtained in closed form by Laplace transform technique. The expressions for the shear stress, rate of heat transfer and rate of mass transfer at the plate for both ramped temperature and isothermal plates are derived. Srinivasacharya and Khaladhar [10]

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described the effect of mixed convection flow of couple stress fluid in a Non-Darcy porous medium with Soret and Dufor effects. The dimensionless governing non-linear partial differential equations for this investigation are solved numerically using finite element method. The effect of the various dimensionless parameters entering into the problem on the velocity, temperature and concentration profiles across the boundary layer is investigated through graphs.

In the present research, the study is to extend the work of Pop *et al.* [8] by introducing suction and injection parameters such as magnetic field parameter and considering symmetric wall temperature and concentration equations on hydromagnetic mixed convection flow of an exothermic fluid in a vertical channel.

2. **FORMULATION**

Let us consider the free convective heat mass transfer MHD flow of a viscous incompressible fluid in a vertical channel formed by two infinite vertical parallel Plates in the presence of chemical reaction. A magnetic field of uniform strength is applied transversely to the plate. The induced magnetic field is neglected while the convection current is induced due to both the temperature and concentration differences. Direction which is taken to be vertically up ward along the channel walls and y-axis is taken to be normal to the plates that are L distance apart. The governing equations in dimensional form can be written as,

V d Udy^2 2'' □□□*B u*0*v*2 *g T T*□('□ □0') *g C C*□('□ 0') □— —□1 *dpdx*' □ 0 (1)

—— □*d Tdv*2 2'—*RT*□*E*' □ 0 (2)

□*Qk ae*0

□^{2}*C* '

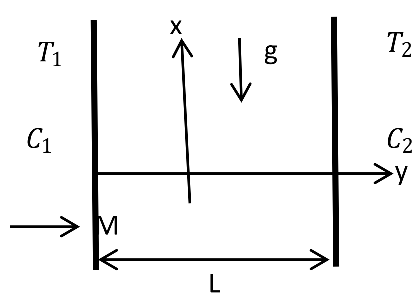
D ——*y*2 ' □ 0 (3)

□

Subject to boundary conditions

$U(0) = 0, U(L) = 0, T(0) = T_1, T(L) = T_2, C(0) = C_1, C(L) = C_2.$ (4)

Where Q is the exothermic factor, ρ is the density, V is the kinematic viscosity, β Is the Coefficient of thermal expansion, g is the acceleration of gravity, α is the thermal diffusivity, D is the effective diffusion coefficients, T_0 is the reference temperature and C_0 is the reference concentration.



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Physical geometry and coordinate system

Then we introduce the following dimensionless variables.

x' y' U' p' ——, U □ $\frac{u\nu}{g\beta T(T_w □ T_0)}$

—— — X □ — , Y □ , $U\,y()$ □ , $P\,x(\,)$ □

2

Re L L U_0 □U_0 $\frac{Kl^{*2}_c}{\nu}$, □ □ $\frac{T□T_0'}{RT_0^2/E}$

C□ $Rc\;cc$ E□$_2$ $_{0'}$,

Re□$U\,L$___V_0,

0

M $_2$□□_____$B\,D_{02}$ $_2$, K_c □

(5)

□

Substituting (5) into (1) to (3) we have the following differential equations

—— $d\,U_{2\,2}$ □ ^{2} NC□□□ (6)

$M\,U$□□□□ □

dy

d_2□ □

___^{2} □Ke □ 0 (7)

dy

d

C^2

___^{2} □ 0 (8)

dy

Subject to the following boundary conditions

U = 0, □ = γ_t, C = γ_c, Y=0, (9)

U = 0, □ = - γ_t, C = - γ_c, Y=1

where λ, N, γ, K, γ_t, γ_c, M, U and Kc are mixed convection parameters, sustention parameter, pressure term, Frank-Kamenetskii number, symmetric wall temperature and concentration, Magnetic field parameter, dimensionless velocity component and chemical reaction parameter respectively which are defined as

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Gr dp T Kh^*, T_1 0, c RC C_1C

E_{020}, K_c 2 v

, , t RT E_{02}

Re dx $B_0^2 h^2$

K EQK $aLRT_{020}$ 2 $e^{E RT_0}$, N Tc((T Tc c_{ww})), U

g h T T 2u v('w 0) M (10)

3. METHOD OF SOLUTION

Equations (6) to (8) subject to (9) are solved analytically by using DTM. The expression of velocity, temperature and concentration respectively are displayed as follows From equation (7)

"Ke 0 (11)

From equation (11), " Ke

$(k+1)(k+2)\,G(k+2) = \dfrac{R(1)^k}{k!}$ (12)

$G(k+2) = \dfrac{R(1)^k}{(k+1)(k+2)k!}$ (13)

Where = G(k), G(0) = γ_t, G(1) = A.

$G(k+2) = \dfrac{R}{(k+1)(k+2)\,k!}$ (14)

From equation (14) when k = 0 and substituting R with K

$G(2) = \dfrac{R}{2!} \quad \dfrac{K}{2!}$ (15)

When k = 1,

$G(3) = \dfrac{R}{(2)(3)1!} \quad \dfrac{R}{3!} \quad \dfrac{K}{3!}$ (16)

When k = 2,

$$G(4) \square \frac{\square R}{(3)(4)2!} \square \frac{\square R}{4!} \square \frac{\square K}{4!} \qquad (17)$$

When k = 3,

$$G(5) \square \frac{\square R}{(4)(5)3!} \square \frac{\square R}{5!} \square \frac{\square K}{5!} \qquad (18)$$

Therefore, By Generalization; $\square\square \sum_{k \square 0} G(k) Y^{k}$ (19)

Then, $\square = G(0) + G(1)Y + G(2)Y^2 + G(3)Y^3 + \ldots$ (20)

$$\square \; \square\square \; \square \; \square_t Ay \square \frac{R}{2!} y^2 \square \frac{R}{3!} y^3 \square \frac{R}{4!} y^4 \square \frac{R}{5!} y^5 \square \ldots \qquad (21)$$

From equation (9) where $\square = -\gamma_t$ at y = 1.
Hence,

(22)

We have $\square \; \square\square \; \square \; \square_t Ay \square \frac{K}{2!} y^2 \square \frac{K}{3!} y^3 \square \frac{K}{4!} y^4 \square \frac{K}{5!} y^5$

From equation (8)

$$\frac{d\, C^2}{dy^2} \square 0$$

Using $B(0) = \gamma_c$, $B(1) = D$, $C(1) = -\overset{\square}{\gamma_c}$

$C'' \square 0$ (23)

$(k\square 1)(k\square 2)\,(B\,k\square \; \square 2)\; 0$ (24)

$B\,k(\square \; \square 2)\, 0$ (25)

When k = 0 from equation (23)

$B(2)$ 0 (26)

When k=1, $B(3)$ 0 (27)

When k=2, $B(4)$ 0 (28)

When k =3, $B(5)$ 0 (29)

Generally; $C\ Y(\ )\ \ B\ k\ Y(\ \ )^k$, $k\ 0$ (30)

$C(Y) = B(0) + B(1)Y + B(2)Y^2 + B(3)Y^3 + \ldots$ (31)

$C\ Y(\ )\ \ {}_c DY$ (32)

From equation (6)

$\frac{d\ U^2 M\ U^2\ \ NC}{dy^2}$

$U''\ \ NC\ MU\ ^2$ (33)

Using F(0) = 0, F(1) = H, U(1) = 0 and let C = B(k), = G(k).

$(k\ 1)(k\ 2)\ (F\ k\ \ 1)\ \ NC\ M\ U\ ^2$ (34)

$F\ k(\ \ 2) = \frac{\ NC\ M\ U^2}{(k\ 1)(k\ 2)}$ (35)

$F\ k(\ \ 2) = \frac{\ G\ k(\ )\ NB\ k(\ )\ M\ U^2}{(k\ 1)(k\ 2)}$ (36)

When k = 0, from equation (36)

$F(2)\ \ G(0)\ NB(0)\ M\ U_2\ \ {}_t N_c\ M\ U_2$ (37)

2 2

When k = 1,

F(3)   A ND M U   2

(38)

6

When k = 2,

   G(2) NB(2) M U²

F(4)

12

K

F(4)   2   N(0)   M U₂   K 2   M U₂

(39)

12 12

When k = 3,

K

20 20 20

Generally, $U(Y) = \sum_{k=0}^{\infty} F(k) Y^k$ (41)

U(Y) = F(O) + F(1)Y + F(2)Y^2 + F(3)Y^3 + F(4)Y^4 + F(5)Y^5+ … (42)

F(5)   G(3)NB(3)MU₂   6 N(0)MU₂ K6 MU₂

(40)

   NᶜM U² ₂ A ND M U   ₂Y₃

K M U₂

U 0 HY 2Y 6   212

Y₄

K

6 M U₂

Y₅

20

(43)

Therefore,

K

12

2

6

K M U₂

6

Y₅

20

(44)

t N c M U² Y₂ A ND M U ₂ Y₃ 2 M U₂ Y₄

U HY

Influence of Skin friction, Nusselt number and Sherwood number

From equation (44) which is the velocity, we obtain the expression of Skin friction as shown below.

2

U HY t N2 c M U₂

Y₂ A ND M U 6 ₂ Y₃ K12 M U₂ Y₄

K M U₂

6 Y₅

20

K

dU H 2 $_t N2_c$ $M\ U_2$ Y 3 $A\ ND\ M\ U$ 6 $_2$ $_2$4 212 $M\ U_2$ Y_3

Y

dy

$\overline{K}6$ $M\ U^2$ Y_4 (45)

5

20

dU

$\frac{dU}{dy}\Big|_{y=0} = H$ (46)

$K\ M\ U_2$

dU H $_t N_c$ $M\ U_2$ $A\ ND\ M\ U$ 2 $_2$

23

$dy_{y\,1}$

$\overline{K}6$ $M\ U_2$ (47)

6

From equation (22) we obtain the rate of heat transfer (Nusselt Number) at the plate y = 0 and y = 1, as follows

$K^2 K y^3 K y^4 K y^5$

$_t Ay\ y$

$$\frac{d\theta}{dy} = A + 2\frac{K}{2!}y + 3\frac{K^2}{3!}y^2 + 4\frac{K}{4!}y^3 + 5\frac{K}{5!}y^4 \quad (48)$$

$$\frac{d\theta}{dy} = A + Ky + \frac{K}{2}y^2 + \frac{K}{6}y^3 + \frac{K}{24}y^4 \quad (49)$$

$$\left.\frac{d\theta}{dy}\right|_{y=0} = A \quad (50)$$

$$\left.\frac{d\theta}{dy}\right|_{y=1} = A + K + \frac{K}{2} + \frac{K}{6} + \frac{K}{24} \quad (51)$$

From equation (32) we obtain the rate of mass transfer (Sherwood Number) at the plate y = 0 and y = 1, as follows

$$C(Y) = \gamma_c + DY$$

$$\frac{dC}{dy} = D \quad (52)$$

$$\left.\frac{dC}{dy}\right|_{y=0} = D \quad (53)$$

$$\left.\frac{dC}{dy}\right|_{y=1} = D \quad (54)$$

4. RESULT AND DISCUSSION

The ordinary differential equations (6 to 8) Subject with boundary conditions (9) are solved analytically using differential transformation method to obtain the equations of temperature, concentration and velocity some of the nondimensional parameter that govern the flow are (λ) is the mixed convection parameter, (γ) is the constant pressure gradient, (γ_t) is the constant temperature parameter, (γ_c) is the wall concentration parameter, (M) is the magnetic field parameter, (U) is the dimensionless velocity component, (N) is the sustentation parameter and (K) is the Frank-Kamenetskii number. For the purpose of discussion the following parameters are fixed throughout the calculation except where otherwise stated

$K = 0.1$, $K_c = 0.1$, $\gamma_c = 0.1$, $\gamma_t = 0.1$, $\lambda = 100$, $\gamma = 0.1$, $M = 0.1$, $U = 0.1$, $N = 0$

The velocity profiles are illustrated in Figure 1 to 3 for different values of ($\gamma_t = 0.1, 0.5, 1.0, 1.5$), (M = 0.1, 1, 1.5, 2), (K = 0.1, 0.5, 1, 1.5) while temperature and concentration profile are illustrated in Figure 4. 2. 4 to 4. 2. 5. For different values of (γ_t = 0.1, 0.5, 1, 1.5), (γ_c =0.1,0.5,1.0,1.5).

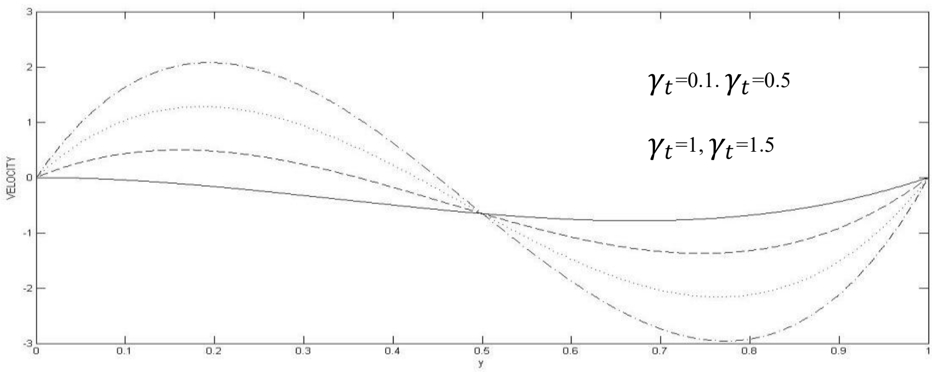

Figure 1. Velocity profile with different values of γ_t

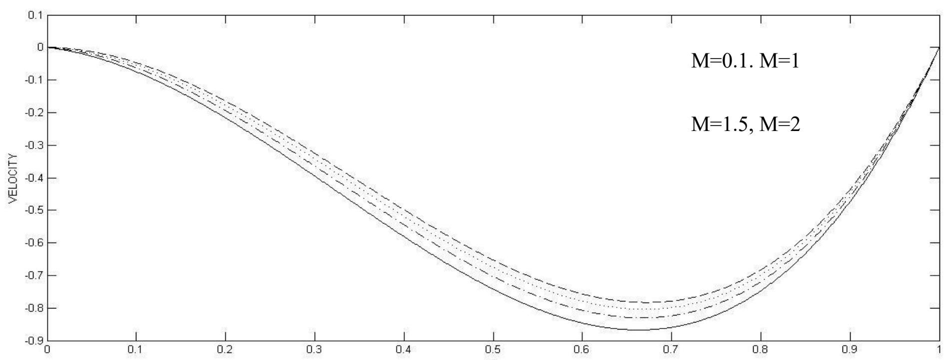

Figure 2. velocity profile with different values M

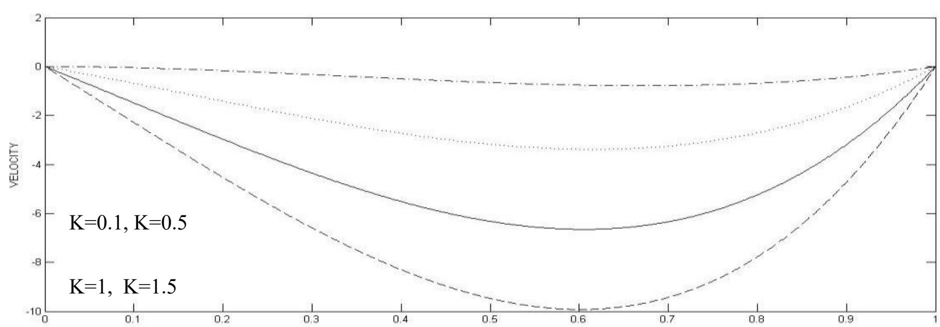

Figure 3. Velocity profile with different values of K

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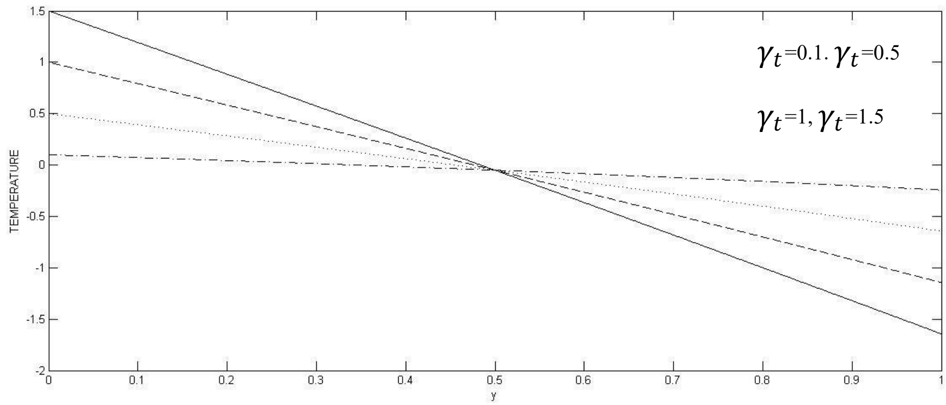


Figure 4. Temperature profile with different values of γ_t

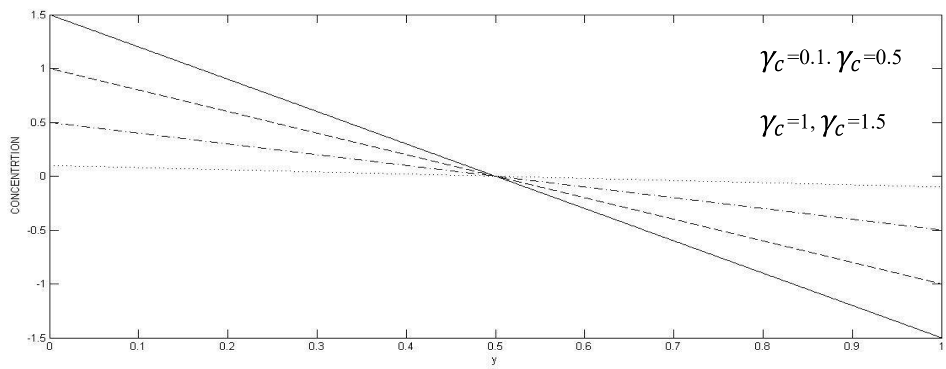


Figure 5. Concentration profile with different values of γ_c

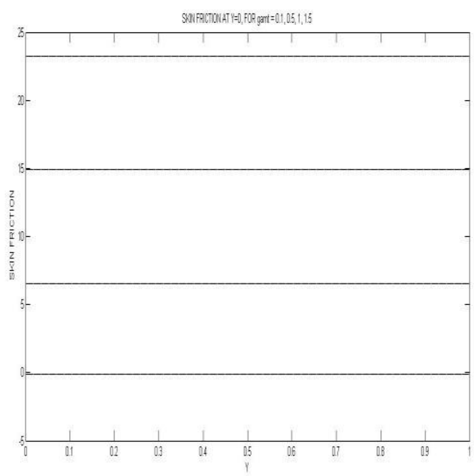


Figure 6a

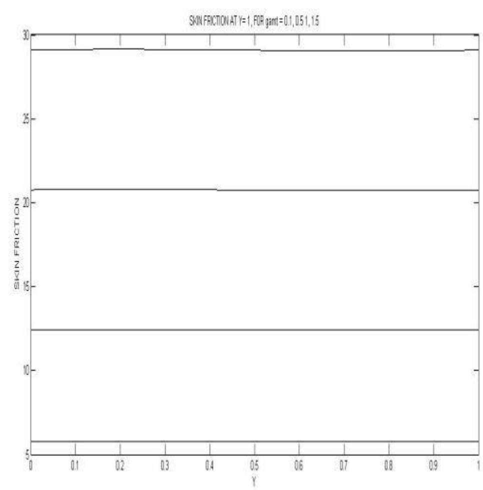


Figure 6b

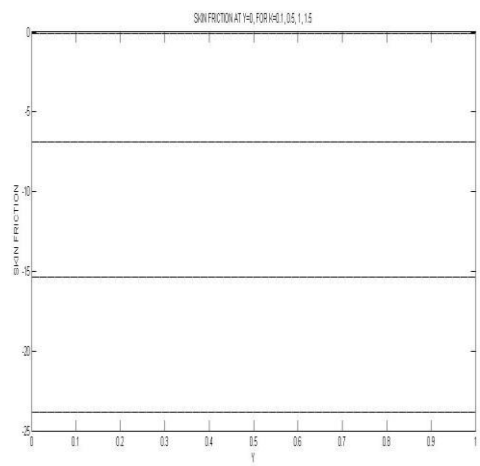


Figure 6c

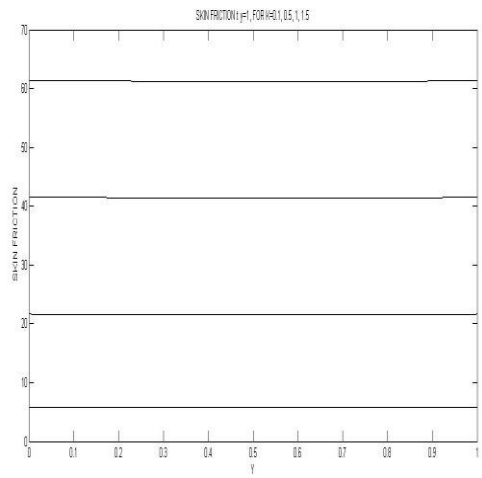
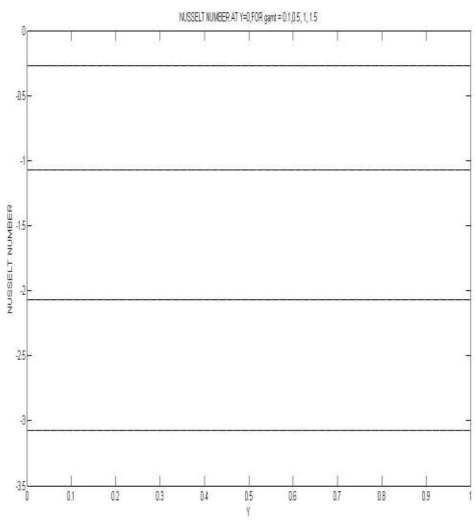


Figure 6d



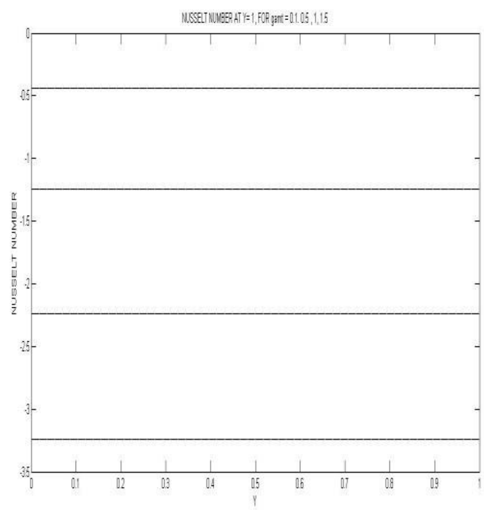


Figure 7b

Figure 7a

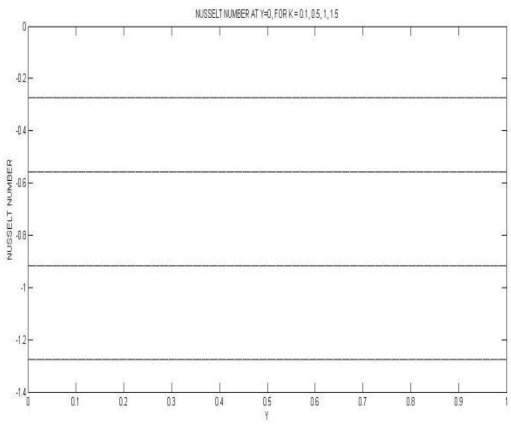


Figure 7c

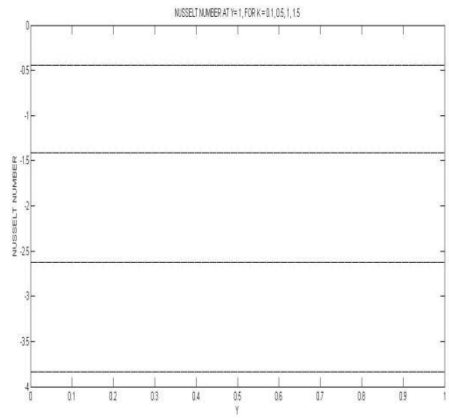


Figure 7d

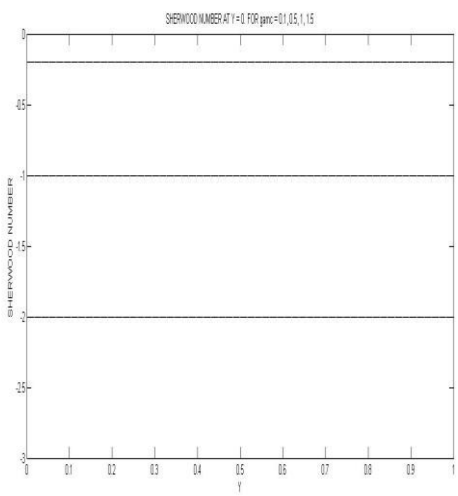


Figure 8a

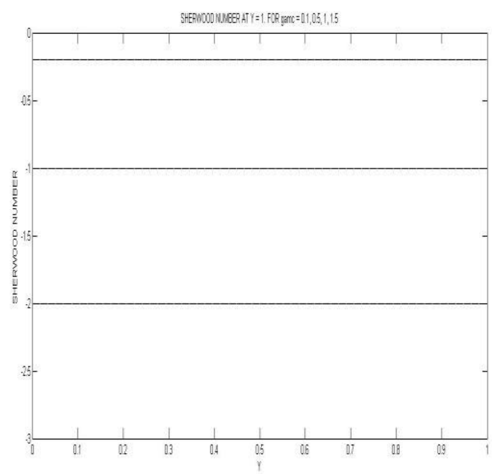


Figure 8b

Figure 1 to 4 exhibit the influence of (Υ_t), (M) and (K) on temperature and velocity profiles, it is observed from Figure 1 that increasing Υ_t accelerate the velocity of the fluid while Figure 2 and 3 shows that increasing M and K lead to significant decrease in velocity of the fluid, it is also observed that there is no point of stagnation when increasing M and K. From Figures 4 and 5 as expected since an increase in Υ_c and Υ_t lead to the significant increase in the strength of the reaction and heating of the channel wall. Thus, correspondingly increases the temperature within the fluid and consequently increases the fluid concentration. The effect of K, Υ_t and Υ_c, on Skin friction, rate of heat (Nusselt number) and mass transfer equation (Sherwood number) are provided on Figure (6 to 8) it is interesting to note from Figure (6a and 6b) that the Skin friction increases with increasing values of wall temperature (Υ_t). It is evident from Figure 6a at the plate y = 0 that the fluid flow is increasing faster and thicker than the fluid flow at the upper plate when y = 1. Figure

(6c and 6d) show the effect of Skin friction with different values (K) at y = 0 and y =1 respectively. It is observed that, the Skin friction decreases with the increasing values of (K) at y = 0 and increases at y = 1. While Figure (7) show the effect of (K) and (γ_t) on the rate of heat transfer (Nusselt number) it is noticed that Nusselt number decreases with increasing (γ_t) and (K) at both lower and upper plate i.e when y = 0 and y = 1. From Figure (8) it show that Sherwood number decreases at y = 0 and y = 1 with increase in γ_c.

5. Conclusion

Hydromagnetic mixed convection flow of an exothermic fluid in a vertical channel is investigated. The numerical results are presented graphically and discussed with various physical parametric values. It was found in this research work that, there is an excellent agreement with Pop *et al.* [13].

It is found that multiple solutions exist for velocity, temperature and concentration. From the research work conducted, it is concluded that:

i. The temperature increases with the increase in γ_t. ii. The concentration increases with increase in γ_c.

iii. The velocity increases with increase in γ_t while increase in K and M suppresses the velocity of the flow.

iv. The effect of γ_t, is to increase the Skin friction while K increases it at upper plate and suppresses it at lower plate. Morealso, the effect of γ_t and K is to decrease the Nusselt number. And increse in wall concentration parameter γ_c decreases Sherwood number.

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