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Recreational Mathematics: Tessellation as an Innovative Pedagogical Approach

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Abstract

Mathematics is the basis of all education, intellectual growth, and problem solving. The failure rate in mathematics education has been exceptionally high because of rote learning. This paper analyses the concept of tessellation as an innovative teaching technique, highlighting its potential to turn mathematics into an enjoyable and captivating subject. The application of tessellation (a teaching method that employs the use of interlocking geometrical shapes) helps in the cultivation of creativity, analytical skills, and a better understanding of mathematics. Tessellation as a teaching technique is supported by contemporary scientific research, especially its combination with digital tools such as GeoGebra. Recent quasi-experimental studies have shown that tessellation software implemented on digital platforms significantly increases the geometric knowledge of elementary school students, as the experimental groups outperform control groups after post-test evaluations. Spatial interventions using tessellation activities increase spatial reasoning abilities and improve mathematics results. This paper reviews the theoretical background, applications in teaching and the impact of tessellation on student performance.

Keywords: *Tessellation, Mathematics Education, Recreational Mathematics, Pedagogical Innovation, Student Engagement, Spatial Reasoning*

INTRODUCTION

In education, mathematics is a key element, forming the basis of analytical and problem-solving skills. Teaching these mathematical skills has primarily been dominated by old-school methods, which are mostly merely dependent on cramming information. These methodologies have indirectly contributed to large-scale student disengagement and poor academic performance. The abstract nature of mathematical concepts, divorced from tangible and relatable contexts, can lead to "math anxiety" and a fixed mindset, where students believe their mathematical ability is innate and unchangeable.

This paper provides a conceptual and empirical synthesis of tessellation as an innovative pedagogical tool in mathematics education. Its objective is twofold: first, to examine the theoretical foundations, types, and educational applications of tessellation across primary, junior-high, and high-school levels; and second, to synthesize contemporary empirical evidence demonstrating how tessellation-based instruction particularly when integrated with digital technologies such as GeoGebra enhances student engagement, spatial reasoning, geometric understanding, and mathematical confidence. The paper adopts a narrative review format, drawing on recent quasi-experimental studies, intervention research, and qualitative case studies to build a coherent argument for tessellation as a transformative pedagogical approach.

The relationship between traditional rote teaching, spatial anxiety, and fixed mindsets warrants explicit examination, as it forms the logical foundation for why tessellation is a necessary pedagogical intervention. Traditional mathematics instruction, characterized by memorization of procedures and formulas divorced from spatial and tangible contexts, creates a cascade of negative cognitive and affective outcomes. When students are required to memorize abstract rules without experiencing the spatial relationships that underlie them such as how shapes fit together, how transformations operate, or how symmetry manifests they are denied the concrete, embodied experiences that build genuine mathematical understanding. This deprivation has two interrelated consequences.

First, it produces spatial anxiety: a discomfort or lack of confidence when confronted with geometric or spatial tasks. Because students have not developed spatial reasoning through hands-on manipulation of shapes, they perceive spatial problems as alien and threatening rather than as intuitive extensions of their physical experience of the world. Second, repeated failure to understand abstract mathematical content reinforces a fixed mindset; the belief that mathematical ability is innate and unchangeable. Students who struggle with rote memorization conclude that they "are not math people," internalizing failure as a permanent trait rather than a temporary state remediable through different learning experiences.

Tactile tessellation practices directly dismantle both spatial anxiety and fixed mindsets through a distinct mechanism: embodied cognition. When students physically cut, fold, rotate, reflect, and translate paper shapes to create interlocking patterns, they are not merely learning geometric vocabulary, they are experiencing mathematical transformation through their hands. The tactile feedback of a shape that does not quite fit, followed by the deliberate adjustment of angle or position, teaches students that mathematical problems are solvable through experimentation and revision. This trial-and-error process, embedded in a creative and playful context, reframes failure as a natural part of discovery rather than evidence of innate incapacity. As students successfully create tessellations, first with guidance, then independently they accumulate direct evidence of their own mathematical agency. This evidence directly

contradicts the fixed mindset narrative, replacing it with a growth-oriented understanding: mathematical ability is developed through active engagement, not inherited.

Furthermore, the spatial reasoning developed through tessellation activities addresses the root of spatial anxiety by building competence and confidence simultaneously. Each successful tiling pattern reinforces the student's sense of spatial efficacy. The hands-on, student-centred nature of tessellation; whether through paper cutting, pattern blocks, or digital tools like GeoGebra, creates a low-stakes environment where experimentation is encouraged and errors are informative rather than punitive. In this way, tactile tessellation practices function as a pedagogical antidote to both spatial anxiety and fixed mindsets, dismantling them through direct, positive, embodied mathematical experience.

In response to these challenges, educators are increasingly seeking more dynamic and engaging methodologies. Recreational mathematics has begun to take the stage in encouraging teachers to give students learning experiences that stimulate motivation. Singmaster (2022) provides a comprehensive overview of the utility of recreational mathematics, highlighting its role in fostering mathematical thinking and engagement. Mulligan (2015) emphasizes the fundamental role of spatial ability and visual thinking skills in young children. Vega Vilca et al. (2025) found that recreational games workshops produced significant improvements across cognitive, procedural, and attitudinal dimensions of academic performance in primary school students. The EDUMAT UIS research group demonstrated the effectiveness of recreational mathematics approaches with 163 fourth- and fifth-grade students from 37 educational institutions, using playful activities to stimulate reasoning and creativity (Universidad Industrial de Santander, 2025).

Tessellation is one such way to repeat a symmetrical pattern in every direction so as to cover a plane without gaps or overlaps. Eberle (2015) defines a tessellation as a pattern of geometric shapes that fulfils three conditions: no gaps exist between the shapes, the shapes do not overlap, and the pattern can go on forever in all directions. The act of designing tessellations encourages students to play around with geometric shapes and transformations such as rotation, reflection, and translation. Their understanding of geometry is enhanced, and it invariably engenders creativity as well as critical thinking. Moreover, tessellations are relevant to many real-world contexts from art and architecture to nature and design making the experience better for the learner.

This paper presents tessellation as a teachable tool for mathematics. It seeks to show inspiring engagement for students through projects and inquiry-based activities concerning tessellation that develop an appreciation for mathematical concepts along with fostering skills that are vital for success in mathematics and other areas of life.

Conceptual and Historical Foundations of Tessellation

The term tessellation refers to the permutation of similar geometric shapes entangled with one another without any gap or overlap. The phenomenon can be found in nature and man-made structures, much like various other areas of investigation such as mathematics, art, and architecture. In mathematics, it is geometry dealing primarily with properties and patterns of tiling.

One of the most discussed figures is Dutch mathematician and artist M. C. Escher, who popularized the idea through visual art. His art represents not only the crisscross of tessellation patterns but goes into deeper mathematical principles like symmetry, transformation, and infinity. Contemporary research continues to explore the pedagogical potential of Escher's

work. Fathauer (2020) provides a comprehensive introduction to tessellations, with a particular focus on "Escheresque" designs. De Almeida Reis and Dias Ferreira Junior (2022) demonstrated that applying Escher's tessellation technique in the high school classroom engaged students in using creativity and logical reasoning, yielding positive qualitative results. Bracher (2025) documented how 9th-grade students learned polygons, angles, symmetry, and transformations through Escher-inspired tessellations, combining computational thinking with art.

Tessellation describes symmetry, transformation, and pattern recognition inherent in tessellations alongside key mathematical concepts such as congruence, similarity, and group theory. This makes tessellations a tool for teaching mathematics whereby students can have visual and tactile experiences of complex mathematical concepts. Examining how different shapes fit together and how they transform into new geometric patterns results in students knowing about symmetry and geometric relationships, enhancing their overall mathematical thinking.

Types of Tessellations

Aydın-Güç and Hacısalihoğlu-Karadeniz (2020) classify tessellations into three major types: regular tessellations (using only one of three regular polygons: square, triangle, or hexagon), semi-regular tessellations (using more than one different regular polygon), and irregular tessellations (using irregular polygons).

1. **Regular Tessellations**

Regular tessellations consist of only one type of regular polygon. A regular polygon is a shape with all sides and angles equal. There are only three shapes that can create a regular tessellation because their interior angles perfectly add up to 360° at each vertex:

- **Equilateral triangles** (each angle is 60°)
- **Squares** (each angle is 90°)
- **Hexagons** (each angle is 120°)

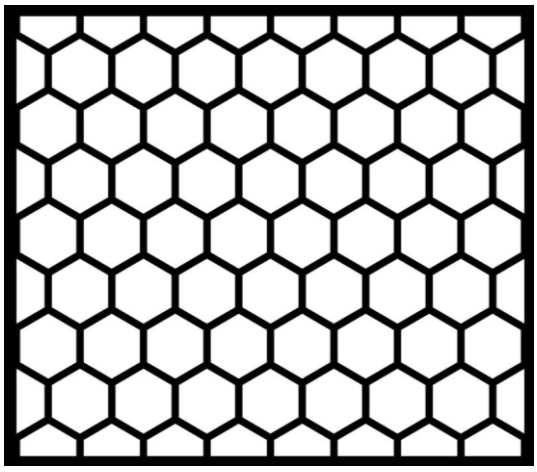


Figure 1: Hexagon tessellation drawing. Adapted from "*Tessellation: What It Is and How to Introduce It to Children,*" by FirstCry Intelli

These tessellations are commonly found in nature and architecture. For example, honeycombs in beehives are hexagonal tessellations, maximizing space efficiency and structural strength (Tressler, 2017).

2. Semi-Regular Tessellations

Semi-regular tessellations involve two or more types of regular polygons arranged in a repeating pattern. The requirement is that the same arrangement of polygons surrounds each vertex. There are exactly eight possible semi-regular tessellations, with combinations such as:

- **Triangles and squares**
- **Hexagons and triangles**
- **Octagons and squares**

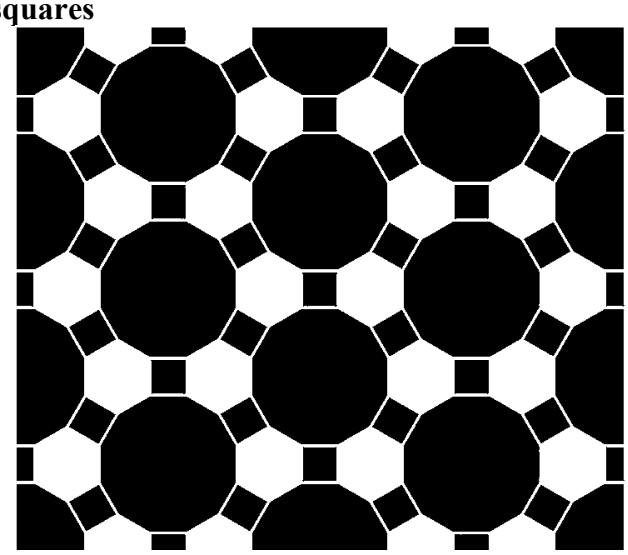


Figure 2: A Semi-Regular Tessellation Drawing. from "*Tessellation: What it is and How to Introduce it to Children,*" by FirstCry Intelli

These tessellations are often seen in decorative tiling, mosaics, and artistic designs. The work of Dutch artist M.C. Escher frequently includes semi-regular tessellations, blending mathematical precision with artistic creativity (De Villiers, 2015).

3. Irregular Tessellations

Irregular tessellations use shapes that are not regular polygons. These can include non-uniform or curved shapes that still fit together without gaps or overlaps. Unlike regular and semi-regular tessellations, irregular tessellations do not have strict geometric rules and often appear in:

- **Nature** (e.g., cracked earth, giraffe skin patterns, fish scales)
- **Art and design** (e.g., Escher's transformational tessellations, puzzle-piece patterns)
- **Architecture** (e.g., stained glass windows, unconventional tiling in buildings)

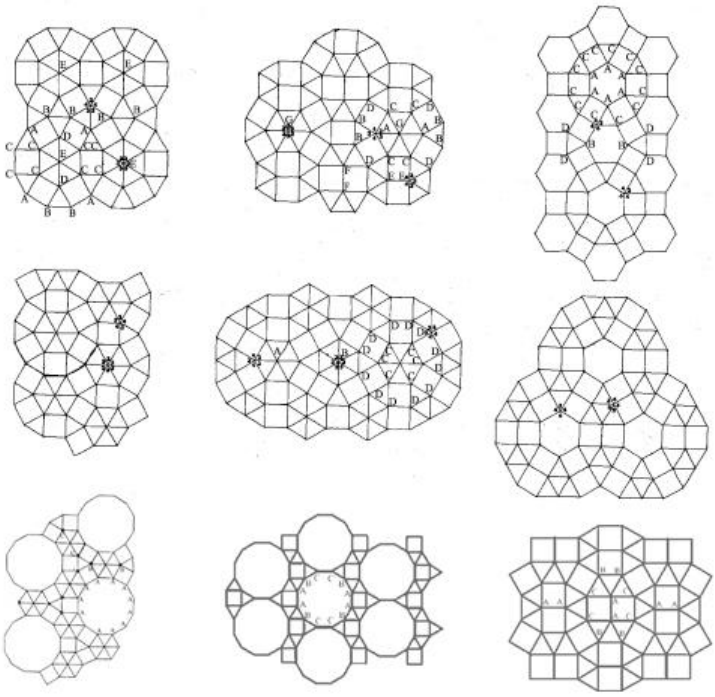


Figure 3: Designs of Irregular Tessellations (Gazi, A., 2010)

Some irregular tessellations can be created by modifying regular polygons through stretching, distorting, or cutting and rearranging parts of the shape while still ensuring a continuous pattern.

Pedagogical Implementation of Tessellation Across Developmental Stages and Modes

The implementation of tessellation-based instruction can be organized along two intersecting dimensions: developmental stages (primary, middle, and high school) and pedagogical modes (manual/tactile, digital/technological, and interdisciplinary/real-world). This section synthesizes research and practical guidance across these dimensions, eliminating redundancy by assigning each cited study to its most relevant context.

Primary School (Grades 1-5): Foundational Spatial Reasoning Through Manual and Digital Modes

At the primary level, tessellation activities introduce basic geometric concepts and spatial reasoning through simple tiling with squares and triangles. The use of these simplest figures introduces foundational ideas about shape properties, congruence, and spatial relationships.

Manual/Tactile Mode: Tangram sets (triangles, squares, and hexagons) and cut-out tiles allow students to create simple shapes and recognize tessellations throughout the classroom. Teachers can add storytelling, where students use tessellated tiles to "construct" towns. Smith and Johnson (2020) note that hands-on activities promote active learning along with better conceptual understanding, while Miller and Davis (2019) observe that cut-and-paste activities allow students to intuitively understand abstract ideas by linking theory to physical objects.

Digital Mode: Scippo, Madiai, and Cuomo (2025) reported that introducing digital tessellation activities to Grade 4 students significantly improved geometry learning outcomes. The experimental group showed significant improvements in post-test scores compared to the pre-test, while the control group did not, supporting the hypothesis that a tessellation-based

program implemented digitally can enhance geometric learning in primary students. A questionnaire revealed high satisfaction with the activities, particularly with the use of a 3D printer and GeoGebra software.

Spatial Reasoning Interventions: Mulligan, Woolcott, Mitchelmore, Busatto, Lai, and Davis (2020) evaluated the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in primary schools, demonstrating that spatial reasoning can be learned and improved with practice, and improvements in spatial reasoning are causally related to improvements in mathematics understanding. The SPAtial Cognition to Enhance mathematical learning (SPACE) programme, as evaluated by Hall, Verdugo, Young, Dowker, and Lewis (2025) with 409 Year 2 pupils, provides robust evidence that structured spatial reasoning activities improve both spatial thinking and mathematics outcomes. Research on second-grade students' spatial modelling abilities (English, 2025) found that young students (n=48, 7-8 years old) displayed adaptive expertise in manipulating shapes to fit along irregular boundaries, demonstrating understanding of shape properties and tessellation principles. Lewis Presser, Braham, and Vidiksis (2025) demonstrated that comprehensive spatial interventions using digital games and hands-on activities effectively enhance preschool spatial skills.

Middle School (Grades 6-9): Semi-Regular Tessellations and Dynamic Geometry

At the middle school level, students explore semi-regular tessellations with pattern blocks and investigate symmetry, angles, and properties of shapes in a more structured yet creative way.

Manual/Tactile Mode: Pattern-block investigations allow students to experiment with semi-regular tessellations, discovering which combinations of regular polygons satisfy the vertex angle sum condition $\sum_{i=1}^{k} \theta_{n_i} = 360°$. Aydın-Güç and Hacısalihoğlu-Karadeniz (2020) found that students' success in different tessellation types may change and called for studies on whether success varies according to tessellation types.

Digital Mode: GeoGebra allows students to manipulate digital shapes and observe the rotations and reflections that generate patterns. Laksmiwati et al. (2023) demonstrated that Techno-based Mathematical Tasks utilizing dynamic geometry software can provide students with opportunities to learn by exploring tessellation activities, contributing to their understanding of geometry concepts. Fitria, Miftah, and Kustiawati (2026) conducted a study at a junior high school in South Tangerang involving 61 students, finding that GeoGebra-assisted explicit instruction significantly improves students' spatial skills by making geometry concepts more accessible and engaging.

Interdisciplinary/Real-World Mode: Having students link tessellations to architecture and nature by taking photos of real examples makes learning more concrete. Silva (2024) implemented a didactic sequence exploring mosaics with regular polygons and Escher's work in a 9th-grade class, highlighting not only student engagement but also the transformation in understanding mathematical concepts through the integration of art, active methodologies, and GeoGebra software.

High School (Grades 10-12): Transformational Tessellations and Advanced Applications

At the high school level, students design original tessellations using transformations and algebraic rules, and engage in problem-solving activities that cross between art (Escher-inspired tessellations) and architecture.

Manual/Tactile Mode: Students create irregular tessellations by applying transformations such as rotation, reflection, and translation, exploring advanced geometric transformation concepts relating symmetry and tiling. Bracher (2025) documented how 9th-grade students learned polygons, angles, symmetry, and transformations through Escher-inspired tessellations, combining computational thinking with art.

Digital Mode: GeoGebra and Desmos provide instantaneous visualization and trial-and-error opportunities to transform abstract transformations into geometry. Costa (2024) showed how GeoGebra enhances teaching of geometric transformations, with students demonstrating learning of translations, rotations, and reflections through tessellation creation. Huang and Fujita (2025) identified moments of shifts in geometric reasoning and spatial skills when grade 7–10 students undertook 3D geometry problems with Virtual Reality, GeoGebra, and tangible artefacts, demonstrating that students' geometric reasoning levels are effectively improved by using multi-duo artefacts. Ndungo, Balimuttajjo, and Akugizibwe (2025) found that Technology-Enhanced Van Hiele Phased Instruction incorporating GeoGebra enhanced visual learning and dynamic interaction with geometric concepts, developing a Geometry Pedagogical Improvement Cycle (GeoPIC) framework.

Interdisciplinary/Real-World Mode: Modesto (2024) conducted research with integrated high school students, observing that the method adopted attracted students' attention to an unknown and fascinating mathematics, as for many students mathematics is reduced to meaningless numerical and algebraic manipulations. Students participated fully in activities including constructions of fractals and Escher's tessellations, with evidence of learning identified through observations and questionnaire analysis. De Almeida Reis and Dias Ferreira Junior (2022) demonstrated that applying Escher's tessellation technique in the high school classroom engaged students in using creativity and logical reasoning, yielding positive qualitative results.

Cross-Cutting Pedagogical Modes

Beyond developmental stages, three pedagogical modes cut across all grade levels:

Manual/Tactile Mode: Drawing equipment (paper, rulers, compasses), cut-and-paste activities, and pattern blocks allow students to explore shapes and patterns directly. These activities promote active learning and conceptual understanding (Smith & Johnson, 2020) and allow students to intuitively understand abstract ideas through linking theory to physical objects (Miller & Davis, 2019).

Digital/Technological Mode: GeoGebra and Desmos facilitate instantaneous visualization, manipulation, and trial-and-error experimentation. Laksmiwati et al. (2023) emphasize that digital technologies are crucial in supporting students in geometry, and their research with dynamic geometry software demonstrated that tessellation activities support diverse learners, including those with low, middle, and high mathematical abilities. Morales Ramírez, Cordero, and Vázquez (2021) found that the visual perception favored by GeoGebra software was essential to develop argumentation processes in high school students when creating semi-regular tessellations. Safitri et al. (2024) empirically demonstrated that dynamic geometry software significantly improves spatial visualization ability. Pinheiro, Alves, and Menezes (2024) demonstrated how geometric visualization supported by GeoGebra software can enhance understanding of mathematical sequences through two-dimensional and three-dimensional visualizations.

Interdisciplinary/Real-World Mode: Recognizing tessellations in architecture, tiling, and nature ties classroom experience with applications outside it. Williams and Harris (2018) noted that real-world connections improve student motivation and understanding of abstract constructs. Dos Santos and Bicudo (2015) demonstrated how dialogue among art and mathematics teachers in continuing education focused on tessellations enabled interdisciplinary courses to be developed, highlighting the constitution of intersubjectivity that enables interdisciplinary work. Oliveira (2025) demonstrated the potential of recreational problems from historical texts like Fibonacci's Liber Abaci for teaching mathematical content, stimulating logical reasoning, creativity, problem solving, and contextualization.

Collaborative and Inquiry-Based Learning

Across all developmental stages, collaborative and inquiry-based approaches enhance tessellation instruction. Students are encouraged to discuss lines of geometric reasoning during group projects. The EDUMAT UIS research group (Universidad Industrial de Santander, 2025) demonstrated that recreational mathematics approaches using playful activities stimulated reasoning and creativity in 163 elementary students from 37 educational institutions. Hall et al. (2025) demonstrated that the SPACE program was implemented with high fidelity in classroom settings, with teachers reporting high acceptability and engagement.

Case Studies in Tessellation-Based Learning

Tessellation-based learning has been shown to significantly contribute to student attention and understanding across multiple educational contexts. In a primary school study, Scippo, Madiai, and Cuomo (2025) found that a digitally implemented tessellation program with 3D printers and GeoGebra software significantly enhanced geometric learning outcomes, with the experimental group showing substantial improvements in post-test scores. The study's findings demonstrated that technological tools like 3D printers and GeoGebra software can be used effectively in primary education, positively impacting geometry learning and promoting high levels of student engagement.

The SPACE programme pilot evaluation by Hall et al. (2025) involved 15 schools with 409 Year 2 pupils and provided robust evidence for the feasibility and promise of structured spatial reasoning interventions. Schools were positive about the self-contained nature of the programme and training, and the resources and support were well received.

Silva (2024) implemented a didactic sequence in a 9th-grade class that explored the intersection of geometry, art, and education through the study of mosaics with regular polygons, highlighting not only student engagement but also the transformation in understanding mathematical concepts.

Impact of Tessellation on Student Performance

Studies reveal that interactive teaching such as tessellation significantly boosts students' acquaintance with mathematics and retention. Scippo, Madiai, and Cuomo (2025) demonstrated that a tessellation-based instructional program supported by digital technologies significantly enhanced geometric learning in primary school students. The experimental group showed significant improvements in post-test scores compared to the pre-test, while the control group did not.

Fitria, Miftah, and Kustiawati (2026) found that the application of GeoGebra-assisted explicit instruction significantly improves students' spatial skills by making geometry concepts more

accessible and engaging. This approach not only enhances conceptual understanding but also fosters interactive learning experiences.

The EDUMAT UIS research group (Universidad Industrial de Santander, 2025) demonstrated that recreational mathematics approaches using playful activities stimulated reasoning and creativity in 163 elementary students from 37 educational institutions. Teachers from participating institutions highlighted the educational value of these types of spaces, which allow for the exploration of new ways of learning outside the traditional classroom.

Research on second-grade students' spatial modelling abilities (English, 2025) found that students displayed adaptive expertise in manipulating shapes to fit along irregular boundaries, demonstrating understanding of shape properties and tessellation principles.

Hall et al. (2025) reported that the SPACE program, implemented with 409 Year 2 pupils, significantly improved both spatial ability and mathematics performance relative to controls. Hidayat et al. (2023) connected visualization skills to mathematical problem-solving, reinforcing the theoretical framework linking spatial skills and cognitive development. Rahman and Utomo (2025) further validated the role of spatial reasoning as a predictor of success in advanced mathematics.

Long-Term Benefits of Tessellation in Education

Unified Analysis of Cognitive, Academic, and Affective Impacts

Tessellation-based learning produces interconnected benefits across three domains: cognitive (spatial reasoning, analytical thinking), academic (mathematical achievement, retention), and affective (confidence, motivation). These domains are not independent but mutually reinforcing. Cognitively, tessellation activities develop spatial awareness through repeated practice with geometric transformations rotation, reflection, and translation building the mental manipulation skills that underpin success in STEM fields (Woolcott et al., 2022; Hall et al., 2025; English, 2025). This spatial competence translates directly into academic gains: Scippo, Madiai, and Cuomo (2025) demonstrated significant post-test improvements in geometry learning, while Fitria, Miftah, and Kustiawati (2026) confirmed that GeoGebra-assisted tessellation instruction enhances spatial skills and conceptual understanding.

Academically, these gains extend beyond immediate performance to retention and problem-solving transfer, as Hidayat et al. (2023) and Rahman and Utomo (2025) linked visualization skills and spatial reasoning to broader mathematical success. Affectively, the hands-on, creative nature of tessellation reduces math anxiety and builds confidence. Anggraeni and Budiharti (2021) found that recreational mathematics activities significantly improved achievement and motivation, while the EDUMAT UIS research group (Universidad Industrial de Santander, 2025) documented heightened engagement and creativity. Crucially, these domains form a feedback loop: spatial competence builds academic confidence, which in turn reduces anxiety and sustains motivation, which further drives cognitive engagement. The creative, low-stakes nature of tessellation tasks reframes failure as experimentation, directly countering the fixed mindsets that traditional rote instruction often produces. Thus, the long-term value of tessellation lies not in any single outcome but in this mutually reinforcing cycle of cognitive growth, academic achievement, and affective resilience.

Implementation Challenges

Despite its promise, tessellation-based instruction faces significant implementation barriers that warrant critical consideration. Resource disparities remain a primary constraint: while

digital tools like GeoGebra enhance tessellation learning (Laksmiwati et al., 2023; Fitria et al., 2026), schools in under-resourced contexts may lack reliable hardware, software licenses, or internet connectivity, potentially widening existing achievement gaps. Curricular time constraints pose a second challenge: tessellation activities are time-intensive, requiring sustained exploration that may conflict with crowded curricula focused on standardized test preparation. Teachers may perceive tessellation as an enrichment activity rather than a core instructional strategy, limiting its integration into regular mathematics lessons. Teacher pedagogical content knowledge represents a third barrier: many educators lack training in spatial reasoning pedagogy and may not recognize how tessellation connects to broader mathematical concepts. Mulligan (2015) and Woolcott et al. (2022) both called for more strategic research agendas and professional development to address this gap. Without adequate teacher preparation, tessellation risks becoming a superficial craft activity rather than a rigorous mathematical experience. These challenges do not negate tessellation's potential but indicate that successful implementation requires attention to resource equity, curricular flexibility, and sustained teacher professional development.

Conclusion

Given that tessellation in mathematics is an innovative pedagogical approach, this paper has attempted to determine the values of tessellation as a potential aid for mathematics education. Moving beyond memorization, tessellation brings a kinetic, engaging shift in the pedagogy of mathematics that nurtures creativity and critical thinking as well as establishes an environment for student learning of important mathematical concepts such as geometry, symmetry, and spatial reasoning. Engaging students through various hands-on activities and real-life applications, tessellation encourages active participation and thus allows for relating the abstract concepts of mathematics much better.

It is also proven through several recent studies that tessellation-based learning not only enhances students' performance in mathematics but also adds to retention, problem-solving capabilities, and spatial awareness, which are crucial for a successful prospect in STEM subjects. As an interdisciplinary medium, tessellation brings mathematics into synergy with applications in art, architecture, and nature, thereby broadening students' horizons with applications of higher significance to mathematics. The cognitive advantages of tessellations include improved spatial reasoning, better analytical skills, and increased confidence in mathematics, showing how special an asset it is in terms of modern-day forward-looking educational methodology. Thus, embracing tessellation in mathematics programs will hold the potential of truly igniting an entire generation of students in understanding the subject deeply in enjoyable ways.

Implications of the Study

The findings synthesized in this paper carry several concise implications for mathematics education:

For Curriculum Designers: Tessellation should be integrated into mathematics curricula as a core instructional strategy rather than an optional enrichment activity, with explicit connections to geometry, symmetry, and transformation standards across grade levels.

For Teacher Education: Pre-service and in-service teacher preparation programs should include training in spatial reasoning pedagogy and tessellation-based instruction, equipping educators with the pedagogical content knowledge to implement these approaches effectively.

For Classroom Practice: Teachers should adopt a blended approach combining manual/tactile activities (paper cutting, pattern blocks) with digital tools (GeoGebra, Desmos) to accommodate diverse learners and resource contexts, while linking tessellation to real-world applications in art, architecture, and nature.

For Policymakers: Investment in digital infrastructure and professional development is necessary to ensure equitable access to tessellation-based learning, particularly in under-resourced schools, to prevent widening achievement gaps.

For Researchers: Further empirical studies are needed to establish causal links between tessellation instruction and long-term mathematical achievement, and to develop validated assessment tools for spatial reasoning outcomes.

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